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Effective supergravity actions for conifold transitions*

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Abstract: We construct gauged supergravity actions which describe the dynamics of M-theory on a Calabi-Yau threefold in the vicinity of a conifold transition. The actions explicitly include N charged hypermultiplets descending from wrapped M2-branes which become massless at the conifold point. While the vector multiplet sector can be treated exactly, we approximate the hypermultiplet sector by the non-compact Wolf spaces $X(1+N)$. The effective action is then uniquely determined by the charges of the wrapped M2-branes.

Keywords: M-Theory, p-branes, Supergravity Models, Supersymmetric Effective Theories.

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Contents

1. Introduction

Conifold transitions are the most studied example of topology change in string theory. But when they first came into the focus of physicists working on Calabi-Yau compactifications [1]–[5] they posed a considerable problem. The mathematics was well understood by that time. As explained in detail in [6], conifold transitions occur when a singular Calabi-Yau space (in the following: CY space) admits two different types of smoothings of its singular points: by either replacing each singularity by a two-sphere (the small resolution) or by replacing it by a three-sphere (the deformation).[1] To make a conifold transition

[1] A more detailed account is given in section 2.

one tunes $N-r$ Kähler moduli of a smooth CY space X with Hodge numbers $(h^{1,1}, h^{1,2})$ such that N two-spheres, $N-r$ of which are independent in homology, shrink to zero volume. The resulting singular CY space $\hat{X}$ is then deformed into a new smooth CY space $\tilde{X}$ by turning on r new complex structure moduli, which control the sizes of the N three-spheres, r of which are homologically independent. The space $\tilde{X}$ has Hodge numbers $\tilde{h}^{1,1} = h^{1,1} - (N-r)$ and $\tilde{h}^{1,2} = h^{1,2} + r$. Many CY spaces are related this way, and a mathematical conjecture, known as Reid's conjecture (or Reid's fantasy) even claims that all CY manifolds form a single connected web [7]. This conjecture has gained much support from string theory, starting from [1]–[5].

Despite their geometrical elegance, conifold transitions originally provided a problem for string theory, because the metric of the moduli space becomes singular at conifold points,[2] and therefore there is no well defined world-sheet CFT corresponding to the singular CY space $\hat{X}$. Likewise, the low energy effective actions (LEEA) S and $\tilde{S}$, which contain the massless modes of string compactifications on the spaces X and $\tilde{X}$, respectively, become singular. Since the conifold points are at finite distance with respect to the metric on moduli space, it seemed that well defined string backgrounds could develop into singular ones within finite time. This changed when the role of branes in string theory was appreciated. As pointed out in [8], the low energy effective action (LEEA) $\tilde{S}$ corresponding to the compactification of the type IIB string on $\tilde{X}$ develops a singularity at the conifold point, which is precisely of the form one gets when integrating out N charged hypermultiplets.[3] Moreover, type IIB string theory indeed contains N charged hypermultiplets which become massless at the conifold point, namely the winding states of D3-branes around the N three-spheres which are contracted. The conifold transition from $\tilde{X}$ to X can then be interpreted as the transition from the Coulomb branch to the Higgs branch of an abelian gauge theory [9]. Conifold transitions work analogously in type IIA string theory and M-theory. Here, the extra hypermultiplets on the Coulomb branch come from D2-branes and M2-branes, respectively, which wrap two-spheres inside X. Thus there is at least a consistent 'macroscopic' description of conifold transitions in terms of four-dimensional, or, in the M-theory case, five-dimensional supergravity. Moreover it is clear that there must exist an extended LEEA $\hat{S}$ which is regular at the conifold point and includes the 'transition states'[4] as dynamical degrees of freedom. The actions S and $\tilde{S}$ corresponding to compactifications on the smooth manifolds X and $\tilde{X}$, respectively, can be interpreted as arising from integrating out the transition states. While the properties of some terms in $\hat{S}$ were discussed in [8, 9], the full action $\hat{S}$ has not been worked out so far. But since all the light modes of a system enter into its low energy dynamics, it is desirable to know $\hat{S}$ explicitly, in order to study the behavior of string compactifications close to a

[2]In this paper, 'conifold point' always refers to the point in moduli space where a CY space becomes singular. The singular points of the CY space are referred to as 'nodes' or 'singular points.'

[3]We always denote a CY space which becomes singular through tuning complex structure moduli by $\tilde{X}$, while we denote a space which becomes singular through tuning Kähler moduli by X. The singular space itself is denoted $\hat{X}$. The corresponding LEEA are $\tilde{S}$, S, and $\hat{S}$, respectively.

[4]The term 'transition states' was coined in [10]. In this paper we will refer to LEEA which explicitly contain transition states as 'extended LEEA.' In [11, 12] this is called the 'in-picture LEEA.'

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conifold transition. Similar remarks apply to other situations in string compactifications where additional light modes occur at special points in the moduli space, in particular to flop transitions in CY compactifications and to compactifications on self-dual circles or tori.

In this paper we continue the work of [10]–[14] on such extended LEEA and construct $\hat{S}$ for conifold transitions in M-theory on CY threefolds. While the vector multiplet sector of $\hat{S}$ can be found exactly for arbitrary conifold transitions, it is currently out of reach to derive the metric on the hypermultiplet manifold from string or M-theory.[5] Therefore we take the simplest family of hypermultiplet manifolds, the non-compact Wolf spaces $X(1+N)$, to model the universal hypermultiplet together with the N hypermultiplets which become massless at the conifold point. M-theory specifies the charges carried by these transition states, and for transitions with $r=1$ and arbitrary N we show explicitly that this determines a unique action $\hat{S}$. We further show that this action has all the properties needed to describe a conifold transition. In particular the model has a non-vanishing scalar potential whose flat directions correspond to a Coulomb and a Higgs branch. Therefore we interprete our model as the leading-order supergravity approximation of the full M-theory LEEA. The physical properties of these actions are investigated in a companion paper [16]. There, we present numerical solutions which undergo dynamical conifold transitions, and we show that the inclusion of the transition states has important consequences for string cosmology and moduli stabilization.

This paper is organized as follows: in section 2 we collect the necessary background material on five-dimensional supergravity, compactifications on CY spaces, and conifold transitions. In section 3 we construct a model with the minimal field content needed to describe a conifold transition. This prepares the stage for a more general model which corresponds to conifold transitions with $r=1$ and arbitrary N in section 4. We present our conclusions in section 5. Some technical details concerning the vector multiplet manifolds are relegated to the appendices A and B. In appendix C we present a particular conifold transition to give the interested reader a more detailed account of the geometrical aspects.

2. Background material

2.1 Five-dimensional supergravity

The LEEA of eleven-dimensional supergravity compactified on a smooth CY threefold X with Hodge numbers $h^{p,q}$ is given by five-dimensional supergravity coupled to $n_V = h^{1,1} - 1$ abelian vector and $n_N = h^{2,1} + 1$ neutral hypermultiplets. When explicitly including the transition states arising in a conifold transition we obtain δn_H additional charged hypermultiplets.

The LEEA which includes these states is given by the general $\mathcal{N}=2, D=5$ gauged supergravity action [17], specialized to the case of n_V vector and $n_H = n_N + \delta n_H$ hypermultiplets. Anticipating the results of the subsequent sections, we limit ourselves to the case of abelian gaugings. The bosonic matter content of this theory consists of the graviton $e_\mu{}^a$,

[5] We refer to [15] for a review of hypermultiplet manifolds in string compactifications.

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n_V+1 vector fields A^I_μ with field strength $F^I_{\mu\nu}=\partial_\mu A^I_\nu-\partial_\nu A^I_\mu$, n_V real vector multiplet scalars ϕ^x, and $4n_H$ real hypermultiplet scalars q^X. The bosonic part of the lagrangian reads:

$$\begin{aligned}\sqrt{-g}^{-1}\mathcal{L}^{\mathcal{N}=2}_{\text{bosonic}} &= -\frac{1}{2}R-\frac{1}{4}a_{IJ}F^I_{\mu\nu}F^{J\,\mu\nu}- \\ &\quad -\frac{1}{2}g_{XY}\mathcal{D}_\mu q^X\mathcal{D}^\mu q^Y-\frac{1}{2}g_{xy}\mathcal{D}_\mu\phi^x\mathcal{D}^\mu\phi^y+ \\ &\quad +\frac{1}{6\sqrt{6}}C_{IJK}\sqrt{-g}^{-1}\epsilon^{\mu\nu\rho\sigma\tau}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\tau-\mathrm{g}^2\mathbb{V}(\phi,q)\,. \end{aligned} \tag{2.1}$$

The scalars ϕ^x and q^X parameterize a very special real manifold $\mathcal{M}_{\text{VM}}$ [18] and a quaternion-Kähler manifold $\mathcal{M}_{\text{HM}}$ [19]. The quaternion-Kähler condition implies that $\mathcal{M}_{\text{HM}}$ is an Einstein manifold. A ghost-free action requires in addition that the Ricci scalar satisfies $\mathcal{R}=-4n_H(n_H+2)$, as is readily seen using the superconformal calculus [20].

The vector multiplet sector is determined by the completely symmetric tensor C_{IJK}, appearing in the Chern-Simons term. This tensor is used to define a real homogeneous cubic polynomial

$$\mathcal{V}(h)=C_{IJK}\,h^I\,h^J\,h^K \tag{2.2}$$

in n_V+1 real variables h^I. The n_V-dimensional manifold $\mathcal{M}_{\text{VM}}$ is obtained by restricting this polynomial to the hypersurface

$$\mathcal{V}(\phi)=C_{IJK}\,h^I(\phi)\,h^J(\phi)\,h^K(\phi)=1\,. \tag{2.3}$$

The coefficients a_{IJ} appearing in the kinetic term of the vector field strength are given by

$$\begin{aligned} a_{IJ}(h) &:= -\,\frac{1}{3}\frac{\partial}{\partial h^I}\frac{\partial}{\partial h^J}\ln\mathcal{V}(h)\Big|_{\mathcal{V}=1} \\ &= -2\,C_{IJK}\,h^K+3\,C_{IKL}\,C_{JMN}\,h^Kh^Lh^Mh^N\,. \end{aligned} \tag{2.4}$$

Defining

$$h^I_x:=-\sqrt{\frac{3}{2}}\frac{\partial}{\partial\phi^x}\,h^I(\phi)\,,\qquad h_I:=C_{IJK}h^Jh^K\,, \tag{2.5}$$

the metric on $\mathcal{M}_{\text{VM}}$ is proportional to the pullback[6] of a_{IJ},

$$g_{xy}(\phi):=h^I_x\,h^J_y\,a_{IJ}\,. \tag{2.6}$$

The hypermultiplet scalars q^X parameterize a quaternion-Kähler manifold of dimension $\dim_{\mathbb{R}}(\mathcal{M}_{\text{HM}})=4n_H$. For $n_H>1$ such manifolds are characterized by their holonomy group,

$$\text{Hol}(\mathcal{M}_{\text{HM}})=\text{SU}(2)\cdot\text{USp}(2n_H)\,, \tag{2.7}$$

while in the case $n_H=1$ they are defined as Einstein spaces with self-dual Weyl curvature. In general quaternion-Kähler manifolds need not be Kähler manifolds or complex manifolds. However, they are quaternionic manifolds, i.e., locally there exists a triplet J^r of almost

[6] The matrix a_{IJ} can be interpreted as a metric on an (n_V+1)-dimensional space, into which $\mathcal{M}_{\text{VM}}$ is immersed by (2.3).

complex structures, which satisfy the quaternionic algebra

$$J^r\, J^s = -\delta^{rs} + \epsilon^{rst}\, J^t\,, \qquad r,s,t = 1,2,3\,. \tag{2.8}$$

In our construction of extended LEEA for conifold transitions we will use the non-compact Wolf spaces

$$X(1+N) = \frac{\mathrm{U}(1+N,2)}{\mathrm{U}(1+N)\times\mathrm{U}(2)}\,, \tag{2.9}$$

as hypermultiplet manifolds. For $N=0$ this series contains the universal hypermultiplet $\frac{\mathrm{U}(1,2)}{\mathrm{U}(1)\times\mathrm{U}(2)}$. To utilize these spaces in the construction of gauged supergravity actions, we need to know the metric, Killing vectors and the associated SU(2)-triplets of moment maps of $X(1+N)$. Using the superconformal quotient construction [21]–[23], these geometrical data have been obtained in [11]. As it will turn out the resulting parameterization is very useful for our construction and we now briefly summarize the relevant results.

We first introduce complex coordinates v^i, u_i, $i=1,\dots,N+1$ with respect to the complex structure J^3.[7] The hypermultiplets of $X(N+1)$ are described by the coordinates v^i, u_i with fixed index i. Using these coordinates we see that the metric on $X(N+1)$ is hermitean with respect to J^3. Its components read[8]

$$\begin{aligned}
g_{u_i\bar u_{\bar\jmath}} &= -\frac{1}{2\phi_-}\left(\eta^{i\bar\jmath} + \bar v^{\bar\jmath}\, v^i\right) + \frac{1}{2\phi_-^2}\left(\eta^{\bar\jmath l}u_l + \bar v^{\bar\jmath}\left(v^l u_l\right)\right)\left(\eta^{i\bar l}\bar u_{\bar l} + v^i\left(\bar v^{\bar l}\bar u_{\bar l}\right)\right),\\
g_{\bar v^{\bar\imath} u_j} &= -\frac{1}{2\phi_-^2}\left(\bar u_{\bar\imath} v^j\left(1+\eta^{k\bar l}u_k\bar u_{\bar l}\right) - \bar u_{\bar\imath}\eta^{j\bar l}\bar u_{\bar l}\left(v^l u_l\right)\right),\\
g_{v^i\bar v^{\bar\jmath}} &= -\frac{1}{2\phi_+}\,\eta_{i\bar\jmath} + \frac{1}{2\phi_+^2}\left(\eta_{i\bar l}\,\bar v^{\bar l}\right)\left(\eta_{\bar\jmath l}\, v^l\right) + \frac{1}{\phi_+\phi_-}\, u_i\bar u_{\bar\jmath} -\\
&\quad -\frac{1}{2\phi_-}\, u_i\,\bar u_{\bar\jmath} + \frac{1}{2\phi_-^2}\, u_i\,\bar u_{\bar\jmath}\left(v^l\, u_l\right)\left(\bar v^{\bar l}\bar u_{\bar l}\right).
\end{aligned} \tag{2.10}$$

Here $\eta_{i\bar\jmath} := \mathrm{diag}\,[-1,\dots-1]$ is minus the $(n-1)\times(n-1)$-dimensional unit matrix, $\eta^{i\bar\jmath}$ denotes its inverse, and ϕ_+ and ϕ_- are given by

$$\phi_+ := 1 + \eta_{i\bar\jmath}\, v^i \bar v^{\bar\jmath}\,, \qquad \phi_- := 1 + \eta^{i\bar\jmath}\, u_i\bar u_{\bar\jmath} + \left(v^i u_i\right)\left(\bar v^{\bar\imath}\,\bar u_{\bar\imath}\right). \tag{2.11}$$

The triholomorphic Killing vectors of the metric (2.10) are completely determined by the generators $[\,t\,]^J{}_I$, $I = i, n, n+1$ of the Lie algebra $su(N+1,2)$ of the isometry group of (2.9). Explicitly they read

$$\hat k^{v^i} = i\, t^i{}_j\, v^j + i\, t^i{}_{n+1} - i\, v^i\, t^{n+1}{}_j\, v^j - i\, v^i\, t^{n+1}{}_{n+1} - \frac{k^\zeta}{2}\left(\eta^{i\bar\jmath}\bar u_{\bar\jmath} + v^i\,\bar v^{\bar k}\bar u_{\bar k}\right), \tag{2.12}$$

$$\hat k^{u_i} = i\, u_i\left(t^{n+1}{}_j v^j + t^{n+1}{}_{n+1}\right) - i\, t^j{}_i\, u_j - i\, t^n{}_i + i\, t^{n+1}{}_i\,\left(v^j\, u_j\right) - 2u_i\, k_z + \frac{\phi_-}{2\phi_+}\, k^\zeta\, \eta_{i\bar\jmath}\bar v^{\bar\jmath}\,,$$

[7] While for a general quaternion-Kähler manifold J^3 does not need to be integrable, it happens to be integrable for $X(1+N)$. In the construction used in [21]–[23] and [11] this is manifest, because $X(1+N)$ inherits a J^3-complex coordinate system from the associated hyper-Kähler cone. Note, however, that the associated fundamental form is not closed, and, therefore, the quaternion-Kähler metric is not Kähler with respect to this complex structure [21].

[8] Note that there is a relative minus sign between the metric obtained from the superconformal quotient construction and the conventions of the supergravity lagrangian (2.1). The results given here are adapted to the supergravity lagrangian.

with k^ζ and k_z given by

$$
\begin{aligned}
k^\zeta &= 2\,i\,\left(t^n{}_i\, v^i + t^n{}_{n+1}\right) \qquad \text{and} \\
k_z &= \frac{i}{2}\left(t^{n+1}{}_i v^i + t^{n+1}{}_{n+1} - t^i{}_n\, u_i - t^n{}_n + t^{n+1}{}_n\, v^i\, u_i\right).
\end{aligned} \tag{2.13}
$$

The components of the Killing vectors with respect to $\partial_{\bar{v}^{\bar{\imath}}}$ and $\partial_{\bar{u}_{\bar{\imath}}}$ are obtained from eq. (2.12) by complex conjugation.

The SU(2)-triplet of moment maps associated with these Killing vectors is

$$
\begin{aligned}
\hat{\mu}^3 &= -\frac{1}{2\phi_+}\left\{\bar{v}^{\bar{\imath}}\,\eta_{\bar{\imath}j}\,t^j{}_k\,v^k + t^{n+1}{}_i\,v^i + \bar{v}^{\bar{\jmath}}\,\eta_{\bar{\jmath}i}\,t^i{}_{n+1} + t^{n+1}{}_{n+1}\right\} + \\
&\quad + \frac{1}{2\phi_-}\Big\{u_i\,t^i{}_j\,\eta^{j\bar{k}}\,\bar{u}_{\bar{k}} + u_i\,t^i{}_n + t^n{}_i\,\eta^{i\bar{\jmath}}\,\bar{u}_{\bar{\jmath}} + t^n{}_n - \left(u_i\,t^i{}_{n+1} + t^n{}_{n+1}\right)\left(\bar{u}_{\bar{\imath}}\bar{v}^{\bar{\imath}}\right) - \\
&\qquad - \left(t^{n+1}{}_i\,\eta^{i\bar{\jmath}}\,\bar{u}_{\bar{\jmath}} + t^{n+1}{}_n\right)\left(u_k v^k\right) + t^{n+1}{}_{n+1}\,\left(u_i\,v^i\right)\,\left(\bar{u}_{\bar{\jmath}}\bar{v}^{\bar{\jmath}}\right)\Big\}\,, \\
\hat{\mu}^+ &= -\frac{i}{2\phi_+^{1/2}\phi_-^{1/2}}\Big\{u_i\,t^i{}_j\,v^j + t^n{}_j\,v^j - t^{n+1}{}_i\,v^i\,\left(u_j\,v^j\right) + u_i\,t^i{}_{n+1} + \\
&\qquad + t^n{}_{n+1} - t^{n+1}{}_{n+1}\left(u_i\,v^i\right)\Big\}\,.
\end{aligned} \tag{2.14}
$$

These are related to the quantities P_I^r appearing in the scalar potential $\mathbb{V}$ of the lagrangian (2.1) by

$$
P_I^1 = \frac{1}{2}\left(\hat{\mu}^+ + \hat{\mu}^-\right), \qquad P_I^2 = -\frac{i}{2}\left(\hat{\mu}^+ - \hat{\mu}^-\right), \qquad P_I^3 = \frac{1}{2}\hat{\mu}^3\,, \tag{2.15}
$$

where $\hat{\mu}^-$ is the complex conjugate of $\hat{\mu}^+$.

Since a conifold transition involves charged hypermultiplets, we must specify an appropriate gauging of the lagrangian (2.1). The scalars take values in a riemannian manifold, and the gauge group must operate on them as a subgroup of the isometry group in order to have an action which is both gauge invariant and invariant with respect to reparameterizations of the scalar manifold. As part of the gauging procedure we need to covariantize the derivatives of the scalar fields with respect to isometries of the vector or hypermultiplet target manifolds,

$$
\mathcal{D}_\mu q^X := \partial_\mu q^X + \mathrm{g}A_\mu^I K_I^X(q)\,, \qquad \mathcal{D}_\mu \phi^x := \partial_\mu \phi^x + \mathrm{g}A_\mu^I K_I^x(\phi)\,. \tag{2.16}
$$

Here the $K_I^X(q)$ and $K_I^x(\phi)$ are the Killing vectors of the gauged isometries in the hypermultiplet and vector multiplet scalar manifold, respectively. As an important consequence of the gauging we now have a non-trivial scalar potential $\mathbb{V}(\phi, q)$. Since we have both vector and hypermultiplets but no tensor multiplets, and since the vector multiplets remain uncharged, the scalar potential is determined by the gauging of the hypermultiplet isometries. In order to write down $\mathbb{V}$ explicitly, we define:

$$
P^r(\phi,q) := h^I(\phi)P_I^r(q)\,, \quad P_x^r(\phi,q) := h_x^I(\phi)P_I^r(q)\,, \quad K^X(\phi,q) := h^I(\phi)K_I^X(q)\,. \tag{2.17}
$$

Here $h^I(\phi)$ are the scalars (2.2), which are associated to the gauge fields A_μ^I, $K_I^X(q)$ denotes the Killing vector of the hypermultiplet isometry for which A_μ^I serves as a gauge connection, and P_I^r is its associated SU(2) triplet of moment maps. The scalar potential takes the form

$$\mathbb{V}(\phi, q) = -4P^r P^r + 2g^{xy} P_x^r P_y^r + \frac{3}{4} g_{XY} K^X K^Y \,. \tag{2.18}$$

To discuss the vacuum structure of this potential, it is further useful to introduce the real 'superpotential' [24]

$$\mathcal{W} := \sqrt{\frac{2}{3} P^r P^r} \,, \tag{2.19}$$

which can be read off from the supersymmetry variations of the gravitino. Under the condition that the phase Q^r, which is defined by

$$P^r = \sqrt{\frac{3}{2}} \mathcal{W} Q^r \,, \qquad Q^r Q^r = 1 \,, \tag{2.20}$$

is independent of the vector multiplet scalars, $\partial_x Q^r = 0$, the scalar potential (2.18) can be rewritten in terms of $\mathcal{W}$:

$$\mathbb{V}(\phi, q) = -6\mathcal{W}^2 + \frac{9}{2} g^{\Lambda\Sigma} \partial_\Lambda \mathcal{W} \partial_\Sigma \mathcal{W} \,. \tag{2.21}$$

Here ϕ^Λ, $\Lambda, \Sigma = 1, \ldots, n_V + 4n_H$ denotes the combined set of vector and hypermultiplet scalar fields and $g^{\Lambda\Sigma}$ is the direct sum of the vector and hypermultiplet inverse metrics,

$$g^{\Lambda\Sigma}(\phi, q) := g^{XY}(q) \oplus g^{xy}(\phi) \,. \tag{2.22}$$

The 'stability form' (2.21) is useful, because it is sufficient to guarantee the gravitational stability of the theory [25].

It was further shown in [24, 26] that the minima of $\mathcal{W}$ are given by the solution of the algebraic equations[9]

$$h_I(\phi_c)\, P^r(\phi_c, q_c) = P_I^r(q_c) \qquad \text{and} \qquad K^X(\phi_c, q_c) = 0 \,. \tag{2.23}$$

Here ϕ_c and q_c denote the restrictions of the scalar fields ϕ and q to the critical submanifolds of the vector and hypermultiplet scalar manifolds where the conditions (2.23) are satisfied. These submanifolds correspond to the supersymmetric vacua of the theory. Indeed, it is straightforward to check that the solutions of the equations (2.23) are critical points of the potential (2.18) with $\mathbb{V}|_{\phi_c, q_c} \leq 0$, implying that the resulting vacua are Minkowski or anti-de Sitter. Further, $\partial_x Q^r = 0$ is always satisfied on the critical submanifolds, so that the restriction of $\mathbb{V}$ to these subspaces can always be rewritten in the stability form (2.21).

In the following discussion, two types of solutions to the equations (2.23) will play a role. First, the conditions (2.23) may fix (some of) the hypermultiplet scalars, while the vector multiplet scalars are unconstrained. This is the Coulomb branch. Second there are solutions where (some of) the vector multiplet scalars are fixed while the hypermultiplet scalars are free to take values in a submanifold of $\mathcal{M}_{\text{HM}}$. This corresponds to the Higgs branch. In between these extreme cases there can also be mixed branches.

[9]For analogous relations in $\mathcal{N} = 2, D = 4$ super-Yang-Mills theory see [27].

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2.2 Calabi-Yau compactifications

When compactifying eleven-dimensional supergravity on a smooth CY threefold X [28], one obtains a five-dimensional ungauged supergravity action, i.e, all fields are neutral under the gauge group $\mathrm{U}(1)^{n_V+1}$ and there is no scalar potential. In this case the objects introduced above acquire the following geometrical interpretation: the vector multiplet scalars encode the deformations of the Kähler class of X at fixed total volume, while the hypermultiplet scalars parameterize the volume of X, deformations of its complex structure, and deformations of the three-form gauge field of eleven-dimensional supergravity. The hypermultiplet containing the volume is called the universal hypermultiplet, because it is insensitive to the complex structure of X.

We need to give some more details. The vector multiplet manifold is closely related to the Kähler moduli space of X, which is a cone, called the Kähler cone. This cone is defined by the condition that the volumes of all curves $C \subset X$, of all holomorphic surfaces $S \subset X$, and of X itself are positive when measured with the Kähler form J

$$\int_C J > 0\,, \qquad \int_S J \wedge J > 0\,, \qquad \int_X J \wedge J \wedge J > 0\,. \tag{2.24}$$

At the boundaries of the Kähler cone some submanifolds of X contract to zero volume and X becomes a singular CY space $\hat{X}$. This will be discussed in the next subsection. To parameterize deformations of the Kähler form J, we introduce a basis C^I, $I = 0, \ldots, n_V$ for the homological two-cycles $H_2(X, \mathbb{Z})$.[10] The dual basis D_I, $I = 0, \ldots, n_V$ for the homological four-cycles $H_4(X, \mathbb{Z})$ is defined by

$$C^I \cdot D_J = \delta^I{}_J\,. \tag{2.25}$$

The Kähler moduli $\hat{h}^I$ are obtained by integrating the Kähler form over the two-cycles C^I:

$$\hat{h}^I = \int_{C^I} J\,. \tag{2.26}$$

This shows that the Kähler moduli are in one-to-one correspondence with the volumes of two-cycles.[11] The volume of X itself is given by

$$\mathrm{vol}(X) = \frac{1}{3!} \int_X J \wedge J \wedge J = \frac{1}{6} C_{IJK} \hat{h}^I \hat{h}^J \hat{h}^K\,, \tag{2.27}$$

where we used Poincaré duality and the definition of the triple intersection numbers

$$C_{IJK} = D_I \cdot D_J \cdot D_K\,. \tag{2.28}$$

Since the modulus corresponding to the volume of X sits in the universal hypermultiplet, we need to disentangle it from the the other Kähler moduli. The rescaled fields

$$h^I = (6\ \mathrm{vol}(X))^{-1/3}\, \hat{h}^I \tag{2.29}$$

[10]Recall that $n_V = h^{1,1} - 1$. The range of I is chosen in order to be consistent with supergravity conventions.

[11]To be precise: $\hat{h}^I$ is the minimal volume which a curve in the homology class C^I can have. This bound is saturated for holomorphic curves.

are precisely the vector multiplet scalars discussed in the last section, and the coefficients of the cubic prepotential $\mathcal{V}(h)$ are precisely the triple intersection numbers. Since the vector multiplet manifold is given by $\mathcal{V}(h) = 1$, we see explicitly that it is the projectivization of the Kähler cone, or, equivalently, a hypersurface corresponding to fixed total volume.

The volume modulus V, which sits in the universal hypermultiplet, is obtained by splitting the volume of X into a dynamical scalar field V and a fixed, dimensionful 'reference volume' v, which relates the eleven-dimensional and the five-dimensional gravitational couplings,

$$\mathrm{vol}(X) = \mathrm{v} \cdot \mathrm{V}\,, \qquad \text{where} \qquad \frac{\mathrm{v}}{\kappa^2_{(11)}} = \frac{1}{\kappa^2_{(5)}}\,. \tag{2.30}$$

Half of the moduli in the remaining $n_H - 1 = h^{2,1}$ hypermultiplets correspond to deformations of the complex structure of X. These moduli are found by integrating the holomorphic $(3,0)$-form of the CY manifold X over the members of a basis of the homological three-cycles. Thus, there is a one-to-one correspondence between complex structure moduli and three-cycles. The other half of the remaining hypermultiplet moduli correspond to deformations of the eleven-dimensional three-form gauge field. The metric on the hypermultiplet manifold is in general only known at tree level. Here one can use the c-map [29] and the whole hypermultiplet manifold is uniquely determined by the special Kähler submanifold spanned by the complex structure moduli of X. However, the hypermultiplet metric is subject to perturbative and non-perturbative corrections. Here only limited results are known, and mostly for the universal hypermultiplet, see [30, 31, 32, 33] and references therein.

2.3 Conifold transitions

We now review the geometry and M-theory physics of conifold transitions. For a more detailed account, see in particular [6] and [34]–[37].

2.3.1 Geometry: local aspects

Conifolds are a particular type of singular CY spaces, which contain a finite number N of isolated nodes (also called double points), which are the simplest singularities of complex threefolds. Let us first discuss the local geometry of a single node. Locally a node can be described as the vertex of a cone in $\mathbb{C}^4$, given by the zero locus of a quadratic polynomial. One way to parameterize this cone is $\Psi = \sum_{a=1}^4 \zeta_a^2 = 0$, where ζ_a are complex coordinates in $\mathbb{C}^4$. By intersecting the locus $\Psi = 0$ with $S^7 \subset \mathbb{R}^8 \simeq \mathbb{C}^4$ one easily sees that $\Psi = 0$ is a real cone with basis $S^2 \times S^3$. This description indicates that the singular point can be smoothed in two different ways, by either keeping the S^2 or the S^3 finite at the tip. The first smoothing is called the small resolution, while the second smoothing, where one replaces the singular point by an S^3, is called the deformation.[12] The process of going from one smoothing to the other describes the local geometry of a conifold transition: one first tunes the Kähler modulus controlling a particular two-cycle to turn it into a node.

[12]The node can also smoothed by replacing it by a submanifold of dimension 1,4 or 5. But this is not compatible with the resulting smoothed manifold being CY.

Then this modulus is frozen in, and the node is deformed into a three-cycle, whose size is controlled by a new complex structure modulus. Obviously, one can also perform this process in the opposite direction, and this is the perspective taken in most of the literature, in particular in [9]. For us it will be convenient to approach the conifold singularity 'from the Kähler moduli side,' because we have full control about the vector multiplet sector, while much less is known about the details of the hypermultiplets.

2.3.2 Geometry: global aspects

Next, let us consider a compact CY space X which develops N nodes when we tune the Kähler moduli such that N isolated holomorphic curves $\mathcal{C}_i, i = 1, \ldots, N$ shrink to points. We will denote the homology classes of these curves by $C^{(i)}$. Naively, one might expect that each node can be resolved or deformed independently. But this is not true, as the condition that the resulting compact smooth manifold is CY imposes constraints (see [6] or [4, 9]). Generically one can either resolve or deform all the nodes simultaneously. Moreover a conifold transition cannot exist if the nodes are independent in homology. Instead, they must satisfies r homology relations ($N > r > 0$)

$$\sum_{i=1}^{N} a_{\rho i} C^{(i)} = 0\,, \qquad \rho = 1, \ldots, r\,. \tag{2.31}$$

Thus the N curves which are contracted to nodes belong to $N - r$ different homology classes and are controlled by $N - r$ Kähler moduli. The special values of these Kähler moduli, where the curves reach zero volume, belong to the boundary of the Kähler cone. The deformation of these nodes results in a new CY manifold $\tilde{X}$, with Hodge numbers

$$h^{1,1}(\tilde{X}) = h^{1,1}(X) - (N - r)\,, \qquad h^{1,2}(\tilde{X}) = h^{1,2}(X) + r\,. \tag{2.32}$$

This shows that in a conifold transitions one looses $N - r$ generators of the two-cycles and gains r generators of the three-cycles, or vice versa. The number of Kähler and complex structure moduli changes accordingly.[13] Note that the Euler number changes: $\chi_{\mathrm{E}}(\tilde{X}) = \chi_{\mathrm{E}}(X) - 2N$.

2.3.3 Physics: microscopic aspects

We now turn to the physical interpretation of conifold transitions in terms of M-theory, which comprises the fundamental or microscopic level of the description. In the smooth CY threefold X the holomorphic curves $\mathcal{C}_i$, with homology classes $C^{(i)}$ can be expanded in the homology basis $\{C^I\}$, so that $C^{(i)} = q^i{}_I C^I$, with integer coefficients $q^i{}_I$. Since the $\mathcal{C}_i$ are holomorphic, they have the minimal volume which is possible in their homology class, $\mathrm{vol}(\mathcal{C}_i) = h^i = q^i{}_I h^I$. Moreover, the N cycles are subject to r homology relations. Therefore the matrix $q^i{}_I$ has rank $N - r$, and the N volumes h^i are controlled by $N - r$ of the $h^{1,1} - 1$ Kähler moduli available at fixed total volume.

[13] While for the small resolution one generically has an overall ambiguity, which leads to two different small resolutions related by a flop, the deformation is unique: even though the deformation contains an entire $SO(2)$-family of three-spheres, these are all homotopically equivalent.

By wrapping M2-branes around these curves, one obtains N hypermultiplets which carry charges $\pm(q^i{}_I), I = 0, \ldots, n_V$, under the gauge fields A^I_μ [35]. The i-th hypermultiplet couples to the particular linear combination $A^i_\mu = q^i{}_I A^I_\mu$. Since the matrix $q^i{}_I$ has rank $N - r$, we see that a subgroup $\mathrm{U}(1)^{N-r}$ of the total gauge group $\mathrm{U}(1)^{n_V+1}$ is gauged.[14] Hypermultiplets are BPS multiplets, and therefore their mass (in suitable units) is given by their central charge, which itself is determined by the electric charges. Explicitly, the central charge of the i-th hypermultiplet is $Z^{(i)} = q^i{}_I h^I$. From the higher dimensional perspective, the mass is given by the dimensional reduction of M2-brane tension $T_{(2)}$ along the curve $\mathcal{C}_i$ [10]:

$$M^{(i)}_{\mathrm{BPS}} = T_{(2)}(6\mathrm{v})^{1/3} \left|q^i{}_I\, h^I\right| . \tag{2.33}$$

Here we indeed recognize the five-dimensional BPS mass formula $M^{(i)}_{\mathrm{BPS}} = \mathrm{const} \cdot |Z^{(i)}|$.[15] At the conifold point $N - r$ Kähler moduli are tuned such that $Z^{(i)} = q^i{}_I h^I = 0$ and the N charged hypermultiplets corresponding to M2-branes wrapped on the curves $\mathcal{C}_i$ become massless.

2.3.4 Physics: macroscopic aspects

Having identified the additional massless states which occur at the conifold point, we now address the question how the transition can be described in terms of a five-dimensional LEEA. Away from the conifold point, when the additional states are heavy, we have compactifications on smooth Calabi-Yau spaces X and $\tilde{X}$. The corresponding effective actions, S and $\tilde{S}$, contain n_V and $\tilde{n}_V = n_V - (N - r)$ abelian vector multiplets and n_N and $\tilde{n}_N = n_N - r$ neutral hypermultiplets, respectively. When we approach the conifold point along the moduli space of X, the N charged hypermultiplets corresponding to M2-brane winding modes become massless. Our goal is to construct a new, extended LEEA $\hat{S}$, which explicitly includes these states. Thus $\hat{S}$ contains n_V vector multiplets and $n_N + N$ hypermultiplets, N of which carry electric charges $q^i{}_I$, as discussed above. By comparing to the spectrum of $\tilde{S}$, one sees that the deformation of $\hat{X}$ into $\tilde{X}$ corresponds to a new branch in the moduli space along which $N - r$ vector multiplets and $N - r$ hypermultiplets become massive. Thus there is a Higgs effect, and the actions S and $\tilde{S}$ are the effective actions of the massless modes on the Coulomb branch and on the Higgs branch of $\hat{S}$, respectively. The Higgs branch is parameterized by the scalars of the r hypermultiplets which remain massless and contain the new additional complex structure moduli of $\tilde{X}$. From the analogous situation in rigid supersymmetric theories one expects that along the Higgs branch the $N - r$ massive vector and hypermultiplets combine into long, non-BPS vector multiplets.[16] The non-BPS na-

[14] The extra U(1) is due to the graviphoton, which belongs to the gravity multiplet.

[15] From the eleven-dimensional point of view the mass of an M2-brane wrapped on $\mathcal{C}_i$ is given by $M^{(i)}_{(11)} = T_{(2)}\,\mathrm{vol}(\mathcal{C}_i) = T_{(2)}(6\mathrm{vV})^{1/3}|q^i{}_I h^I|$. However, the relation between the eleven-dimensional and five-dimensional metrics involves a conformal rescaling by the volume modulus V [11].

[16] See for example [38].We will see later that in our explicit gauged supergravity models the Higgs effect indeed works as in the rigid case.

ture of these states fits very well with the observation that in the microscopic picture there are no BPS branes which could wrap three-cycles in order to give point-like BPS states.[17]

Let us next discuss how the extended LEEA $\hat{S}$ can be found explicitly. As usual with effective actions, S and $\tilde{S}$ need not be obtainable from $\hat{S}$ by simply truncating out the additional modes. In general, integrating out states will lead to threshold corrections, which modify some of the terms involving the other states. This can be seen explicitly in the vector multiplet sector, because this part of the actions $S, \hat{S}$ and $\tilde{S}$ is determined completely by the corresponding homogenous cubic polynomials $\mathcal{V}, \hat{\mathcal{V}}$ and $\tilde{\mathcal{V}}$. For S and $\tilde{S}$ these polynomials are determined by the triple intersection numbers C_{IJK} and $\tilde{C}_{\tilde{I}\tilde{J}\tilde{K}}$ of X and $\tilde{X}$. The polynomial $\hat{\mathcal{V}}$ can be related to $\mathcal{V}$ as follows: one considers the intermediate regime where the transition states are massive, but less heavy than all the other massive states. Then the descriptions in terms of $\hat{S}$ and S are both valid, and one can determine the relative threshold correction between these two effective actions by explicitly integrating out the N charged hypermultiplets. Adapting the result found in [35] we obtain:

$$\hat{\mathcal{V}} = \mathcal{V} - \frac{1}{2} \sum_{i=1}^{N} \left(h^i\right)^3 = \mathcal{V} - \frac{1}{2} \sum_{i=1}^{N} \sum_{I=0}^{n_V} \left(q^i{}_I h^I\right)^3 . \tag{2.34}$$

Note that since $q^i{}_I$ has rank $N-r$, the difference $\hat{\mathcal{V}} - \mathcal{V}$ only depends on the $N-r$ Kähler moduli which control the N curves $\mathcal{C}_i$.

The polynomials $\hat{\mathcal{V}}$ and $\tilde{\mathcal{V}}$ are related by integrating out $N-r$ vector and $N-r$ hypermultiplets. As none of these states is charged under the remaining gauge group $\mathrm{U}(1)^{\tilde{n}_V+1} = \mathrm{U}(1)^{n_V+1-(N-r)}$, there is no threshold correction which modifies the interactions between the remaining vector multiplets, and $\tilde{\mathcal{V}}$ is obtained by freezing the $N-r$ Kähler moduli, which is equivalent to setting $h^i = 0$:[18]

$$\tilde{\mathcal{V}} = \hat{\mathcal{V}}\Big|_{h^i=0} = \mathcal{V}|_{h^i=0} . \tag{2.35}$$

This prescription appears to agree with the relation between C_{IJK} and $\tilde{C}_{\tilde{I}\tilde{J}\tilde{K}}$ predicted by algebraic geometry, where one drops the part of the intersection ring which is generated by the duals of the of the two-cycles $C^{(i)}$.[19]

While the vector multiplet sector can be treated exactly, including threshold corrections, the situation is much less satisfactory in the hypermultiplet sector. As explained above, already the hypermultiplet manifolds of S and $\tilde{S}$ are only known at tree level. In order to determine this manifold for $\hat{S}$, one needs to know in addition the couplings involving the groundstates of the wrapped M2-branes. In M-theory it is currently not known

[17]The exact microscopic origin of these degrees of freedom remains to be clarified. For the case of type IIB string theory it was argued in [39] that these states correspond to massive string excitations.

[18]To obtain $\tilde{\mathcal{V}}$ explicitly, one can apply a linear transformation which block-diagonalizes the charge matrix $q^i{}_I$ such that the $N-r$ Kähler moduli controlling the nodes correspond to, say $h^{I'}$ with $I' = 1, \ldots, N-r$. Then one sets $h^{I'} = 0$. Examples will be given in the next two sections.

[19]Examples for the computation of triple intersection numbers and their behavior in conifold transitions are given in appendix C.

how to compute this. Going to type IIA string theory by dimensional reduction does not help either: here one has techniques such as boundary conformal field theory to handle D-branes, but these can only be applied if the corresponding world-sheet conformal field theory is rational, for example at Gepner points in CY moduli spaces. However, at conifold points one does not even have a well-defined conformal field theory, and effective supergravity is so far the only way to see that the full, non-perturbative string or M-theory physics is nevertheless regular. Because of these difficulties we will not try to determine the exact hypermultiplet manifold from microscopic physics, but instead make a choice, based on simplicity, and take the non-compact Wolf spaces $X(1+N)$ as hypermultiplet manifolds of $\hat{S}$. As we will see this will still pose considerable technical problems, which, however, can be solved. Note that we still have the essential microscopic input, namely the charge matrix ${q^i}_I$, which determines the gauging of $\hat{S}$ in terms of the geometry of the transition. Since $\hat{S}$ is a gauged supergravity action, the choice of the scalar metric and of the gauging determines the whole action, in particular the scalar potential and therefore the mass matrix. Since $X(1+N)$ is a curved quaternion-Kähler manifold, it is not a priori clear whether there exists a gauging which leads to a potential with the structure needed to describe a conifold transition. But in the following sections we will show explicitly that such gaugings exist and that they are uniquely determined by the microscopic data.

3. Actions for conifold transitions: a minimal model

We now have all the ingredients to write down explicitly the LEEA for conifold transitions, including the charged extra states which become massless at the transition locus. To illustrate the key features of the construction, we first consider a model with the minimal number of multiplets before turning to the description of a general conifold transition. In both cases we proceed by first constructing the lagrangian and then showing that the scalar masses obey the conditions arising from the microscopic picture.

Before embarking on the actual construction of the lagrangian, let us comment on the general vacuum structure expected from an effective action describing a conifold transition. From the microscopic picture we expect the vacuum manifold to consist of two branches, a Coulomb branch (corresponding to the compactification on X) where the extra light hypermultiplets are massive and a Higgs branch (corresponding to the compactification on $\tilde{X}$) where some of the vector multiplets become massive and the gauge group is spontaneously broken $\mathrm{U}(1)^{n_V+1} \to \mathrm{U}(1)^{n_V+1-(N-r)}$.

In terms of the algebraic eqs. (2.23) this vacuum structure can be understood as follows. Suppose we gauge an isometry where we can fix some of the hypermultiplet scalars to obtain $P^r_I(q) = 0$ while the corresponding Killing vector $K^X_I(q)$ remains non-zero. Having $P^r_I(q) = 0$ suffices to meet the first condition in (2.23).[20] In order to satisfy the second equation, $h^I(\phi)K^X_I(q) = 0$, we then have the choice to either fix the remaining hypermultiplet scalars

[20]In terms of four-dimensional super-Yang-Mills theory the conditions $P^1_I(q) = P^2_I(q) = 0$ and $P^3_I(q) = 0$ correspond to F- and D-flatness, respectively [27].

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such that $K_I^X(q) = 0$ or the vector multiplet scalars such that $h^i(\phi) = \sum_I q_I^i h^I(\phi) = 0$.[21] The first choice corresponds to the Coulomb branch, while the second one gives rise to the Higgs branch. The intersection locus where both $K_I^X(q) = 0$ and $h^i(\phi) = 0$ are fulfilled is the conifold locus.

3.1 Constructing the action

As is well known from rigid $\mathcal{N} = 2$ supersymmetric theories, the minimal field content for having a Coulomb and a Higgs branch is one vector multiplet together with two charged hypermultiplets. In the effective supergravity framework this field content can be used to describe a conifold transition with $N = 2, r = 1$, if we truncate out all the multiplets which are massless along both branches. In such a transition the Hodge numbers are related by $h^{1,1}(\tilde{X}) = h^{1,1}(X) - 1$, $h^{1,2}(\tilde{X}) = h^{1,2}(X) + 1$, while the Euler number changes according to $\chi_E(\tilde{X}) = \chi_E(X) - 4$. The rank of the gauge group is reduced by one. Besides the gravity multiplet, the minimal model contains only the vector multiplet corresponding to the U(1) which is Higgsed, together with two hypermultiplets. In order to describe a conifold transition, we need the following behavior: along the Coulomb branch the two hypermultiplets must be massive. They must become massless at the conifold point, where one can give a vacuum expectation value to some of the hypermultiplet scalars, with the result that the U(1) is Higgsed. The Higgs branch is parameterized by the scalars in one hypermultiplet, while the vector multiplet and the other hypermultiplet become massive.[22] Note that the universal hypermultiplet has been truncated out together with all the other neutral hypermultiplets. We take the vector multiplet manifold to be the most general one, see appendix A, while the hypermultiplet manifold is $X(2)$.

Geometrically the conifold transition involves two two-cycles $C^{(i)}$, $i = 1, 2$, which are subject to the homology relation $C^{(1)} + C^{(2)} = 0$.[23] The charges of the hypermultiplets are obtained by expanding the $C^{(i)}$ in the homology basis C^I, $C^{(1)} = -C^{(2)} = \sum_{I=0}^{n_V} q^1{}_I C^I$. The volume of $C^{(1)}$ is given by $h^\star(\phi) = \sum_{I=0}^{n_V} q^1{}_I h^I(\phi)$. In order to work with the truncated model with one vector multiplet, we might need to perform a (not necessarily integer valued) basis transformation of the h^I such that $q^1{}_I = 0$ for $I > 1$ and $h^\star(\phi) = \sum_{I=0}^{1} q^1{}_I h^I(\phi)$. The corresponding gauge field is $A^\star_\mu = \sum_{I=0}^{1} q^1{}_I A^I_\mu$. The condition $C^{(1)} + C^{(2)} = 0$ between the two two-cycles translates into the requirement that the two hypermultiplets carry opposite U(1) charges. The Killing vector encoding this charge assignment should then have the form

$$k^X_{\mathrm{Cf}} = i\,\left[v^1, -v^2, -\bar{v}^1, \bar{v}^2, -u_1, u_2, \bar{u}_1, -\bar{u}_2\right]^{\mathrm{T}}\,, \tag{3.1}$$

[21] Since the hypermultiplets are charged with respect to $A^i_\mu = q^i{}_I A^I_\mu$, it is clear from (2.16) that this is sufficient to have $h^I(\phi) K_I^X(q) = 0$.

[22] As we will see the hypermultiplet which parameterizes the Higgs branch is a specific linear combination of the two hypermultiplets which we use on the Coulomb branch, see (3.11), (3.12).

[23] We are not aware of a concrete example of a conifold transition with $N = 2, r = 1$. In appendix C we give an example of a conifold transition with $N = 4, r = 3$ which, by mirror symmetry, maps to a conifold transition with $N = 4, r = 1$. The conifold transition of the quintic has $N = 16, r = 1$. The LEEA constructed in the next section correspond to $r = 1$ with arbitrary N.

with respect to the basis

$$\{\partial_{v^1}, \partial_{v^2}, \partial_{\bar{v}^1}, \partial_{\bar{v}^2}, \partial_{u_1}, \partial_{u_2}, \partial_{\bar{u}_1}, \partial_{\bar{u}_2}\}\,. \tag{3.2}$$

The relative sign in the charge assignment between v^i and u_i stems from the fact that the u_i transform in the conjugate representation of the gauge group [22].

Equating the Killing vector (3.1) with the expression for the most general triholomorphic Killing vector of $X(2)$, (2.12), we find that (3.1) indeed is a symmetry of $X(2)$ which is generated by the Lie algebra generator

$$t_{\mathrm{Cf}} = \mathrm{diag}\,[\,1\,,\,-1\,,\,0\,,\,0\,]\,. \tag{3.3}$$

The SU(2)-triplet of moment maps associated with this isometry is then obtained by substituting this generator into the general expression for the moment maps on $X(2)$, (2.14), and reads

$$\hat{\mu}^r_{\mathrm{Cf}} = \begin{bmatrix} -\frac{i}{2\phi_+^{1/2}\phi_-^{1/2}}\left(v^1u_1 - v^2u_2 - \bar{v}^1\bar{u}_1 + \bar{v}^2\bar{u}_2\right) \\ -\frac{1}{2\phi_+^{1/2}\phi_-^{1/2}}\left(v^1u_1 - v^2u_2 + \bar{v}^1\bar{u}_1 - \bar{v}^2\bar{u}_2\right) \\ -\frac{1}{2\phi_-}\left(u_1\bar{u}_1 - u_2\bar{u}_2\right) + \frac{1}{2\phi_+}\left(v^1\bar{v}^1 - v^2\bar{v}^2\right) \end{bmatrix}, \tag{3.4}$$

where the components of the moment map $\left\{\hat{\mu}^1, \hat{\mu}^2, \hat{\mu}^3\right\}$ are adapted to the basis associated with the complex structures (2.8).

Next we need to identify the gauge connection $A^\star_\mu$. The Higgs branch condition $h^\star(\phi_c)K^X_\star(q_c) = 0, K^X_\star(q_c) \neq 0$ requires that $h^\star(\phi)$ must have a zero for some value ϕ inside the vector multiplet scalar manifold. The discussion in appendix A then fixes $h^\star(\phi) = h^1(\phi)$ since $h^0(\phi)$ does not have this property.

After determining the gauge connection $A^\star_\mu$, the Killing vector k^X_{Cf}, and its associated triplet of moment maps, we now have all the ingredients for gauging the general supergravity lagrangian (2.1). We start with the kinetic terms. Since we do not gauge any isometries of the vector multiplet scalar manifold, the corresponding gauge covariant derivative is just the partial derivative,

$$\mathcal{D}_\mu\phi^x = \partial_\mu\phi^x \qquad \Leftrightarrow \qquad K^x_I(\phi) \quad \text{trivial}\,. \tag{3.5}$$

We now turn to the gauging of the hypermultiplet sector. Since the U(1) gauge connection of the isometry (3.1) is $A^\star_\mu = A^1_\mu$ we must set

$$K^X_0(q) = 0\,, \qquad K^X_1(q) = k^X_{\mathrm{Cf}}(q)\,. \tag{3.6}$$

The covariant derivative for the hypermultiplet scalars then becomes

$$\mathcal{D}_\mu\, q^X = \partial_\mu\, q^X + \mathrm{g}\, A^1_\mu\, k^X_{\mathrm{Cf}}(q)\,. \tag{3.7}$$

This expression explicitly shows that the two hypermultiplets v^1, u_1 and v^2, u_2 carry opposite U(1) charge with respect to the gauge connection A^1_μ. Furthermore, we have included an explicit gauge coupling constant g. At the level of five-dimensional gauged supergravity this is a free parameter, but we will see later that g is fixed by the microscopic data of the full M-theory.

Next we turn to the scalar potential (2.18). Taking into account the relations (2.15) we set

$$P_0^r(q) = 0\,, \qquad P_1^r(q) = \frac{1}{2}\,\hat{\mu}^r_{\rm Cf}(q)\,. \tag{3.8}$$

Correspondingly, P^r is obtained as

$$P^r(\phi,q) = \frac{1}{2}\,h^1(\phi)\,\hat{\mu}^r_{\rm Cf}(q)\,. \tag{3.9}$$

In order to construct the scalar potential, we work out the superpotential (2.19). For the P^r above this is given by

$$\begin{aligned}\mathcal{W} = \frac{h^1}{\sqrt{6}}\Bigg\{&\left(\frac{1}{2\phi_+}\left(v^1\bar{v}^1 - v^2\bar{v}^2\right) - \frac{1}{2\phi_-}\left(u_1\bar{u}_1 - u_2\bar{u}_2\right)\right)^2 + \\ &+ \frac{1}{\phi_+\phi_-}\left(v^1u_1 - v^2u_2\right)\left(\bar{v}^1\bar{u}_1 - \bar{v}^2\bar{u}_2\right)\Bigg\}^{1/2}.\end{aligned} \tag{3.10}$$

From this expression it is then straightforward to check that Q^r defined in eq. (2.20) is independent of the vector multiplet scalar. This implies that the scalar potential $\mathbb{V}(\phi,q)$ is determined by the 'superpotential' $\mathcal{W}$, because it can be brought to the stability form (2.21).

3.2 Vacua and mass matrix

The supersymmetric vacua of this potential are determined by the conditions (2.23). Looking at the moment map (3.4) reveals that $\hat{\mu}^r_{\rm Cf}$ vanishes if and only if $v^1 = v^2, u_1 = u_2$. Restricting the Killing vector (3.1) to this subset we find that the resulting expression is non-vanishing. The second condition in (2.23) can then be met by either setting the hypermultiplet scalars to zero, which corresponds to the Coulomb branch

$$\mathcal{M}_{\rm Coul} = \left\{v^1 = u_1 = v^2 = u_2 = 0\,,\ \phi \text{ undetermined}\right\}\,, \tag{3.11}$$

or fixing $\phi = 0$ which yields the Higgs branch

$$\mathcal{M}_{\rm Higgs} = \left\{v^1 = v^2 =: v\,,\ u_1 = u_2 =: u\,,\ \phi = 0\,\right\}. \tag{3.12}$$

On the latter, the non-trivial vacuum expectation value of the hypermultiplet scalars spontaneously breaks the U(1) gauge group. Restricting the superpotential (3.10) to these vacua, we find that the resulting expression vanishes. This implies that these vacua are Minkowski, establishing that our model has the correct vacuum structure to describe a conifold transition.

To complete our discussion, we also calculate the masses of the fields on the two vacuum branches. For the scalars the masses are given by the eigenvalues of the mass matrix

$$\mathcal{M}^\Lambda{}_\Sigma = {\rm g}^2\; g^{\Lambda\Xi}\,\frac{\partial}{\partial\phi^\Xi}\,\frac{\partial}{\partial\phi^\Sigma}\,\mathbb{V}(\phi,q)\bigg|_{\mathcal{M}_{\rm Vac}}\,, \tag{3.13}$$

where $g^{\Lambda\Sigma}$ has been defined in eq. (2.22). Arranging the coordinates on $\mathcal{M}_{\rm HM}\times\mathcal{M}_{\rm VM}$ as

$$\phi^\Lambda = \left\{\,v^1\,,\,v^2\,,\,\bar{v}^1\,,\,\bar{v}^2\,,\,u_1\,,\,u_2\,,\,\bar{u}_1\,,\,\bar{u}_2\,,\,\phi\,\right\}, \tag{3.14}$$

the evaluation of the mass matrix on the Coulomb branch yields

$$\mathcal{M}^{\Lambda}{}_{\Sigma}\big|_{\rm Coul} = (m_{\rm Coul})^2 \,{\rm diag}\,[\,1\,,\,1\,,\,1\,,\,1\,,\,1\,,\,1\,,\,1\,,\,1\,,0\,]\,, \tag{3.15}$$

with

$$(m_{\rm Coul})^2 = \frac{3}{2}\,{\rm g}^2\,\left(h^1\right)^2\,. \tag{3.16}$$

In terms of the microscopic picture $|h^1|$ corresponds to the volume of the shrinking cycle. This implies that (3.16) has precisely the structure expected from the eleven-dimensional point of view. By comparing with (2.33) and using (2.30) together with the value $T_{(2)} = (\frac{8\pi}{\kappa^2_{(11)}})^{1/3}$ of the M2-brane tension [10], we find that g is fixed by M-theory,

$${\rm g} = \sqrt{\frac{2}{3}}(48\,\pi)^{1/3}\,. \tag{3.17}$$

Thus the extended LEEA is completely fixed once we choose the hypermultiplet manifold to be $X(2)$.

Let us now turn to the Higgs branch. Here we have to evaluate the mass matrix for the scalar fields (3.13) and in addition obtain a mass term for the vector field A^1_μ. In the latter case we note that since $a_{IJ}|_{\phi=0} = \mathbb{1}_{2\times 2}$ (cf. appendix A), the kinetic term for the vector fields has its standard form, so that the mass of A^1_μ can be read off directly from the term proportional to $\left(A^1_\mu\right)^2$ arising in the hypermultiplet kinetic term. Defining

$$(m_{\rm Higgs})^2 = 2\,{\rm g}^2\,\left(\frac{1}{\phi_+}\,v\bar{v} + \frac{1}{\phi_-}u\bar{u}\right), \tag{3.18}$$

where $\phi_\pm$ is understood to be restricted to $\mathcal{M}_{\rm Higgs}$, we find that the vector field A^1_μ, the vector multiplet scalar ϕ and three hypermultiplet degrees of freedom acquire the mass $m_{\rm Higgs}$ while the remaining fields stay massless. In terms of supersymmetry algebra representations (see for example [38]) these modes organize into one massless hypermultiplet and one long vector multiplet of mass $m_{\rm Higgs}$. The latter contains all the massive degrees of freedom and one massless hypermultiplet mode which is the would-be Goldstone boson that provides the longitudinal mode of the vector field. This shows explicitly that the vector multiplet 'eats up' one of the hypermultiplets and becomes a non-BPS multiplet.

4. Actions for conifold transitions: general case

In the last section we illustrated the key features of an extended LEEA describing a conifold transition by means of a minimal model. Now we generalize this setup to the case where N two-cycles satisfying $r = 1$ homology relations contract to nodes. When including the neutral universal hypermultiplet, the hypermultiplet sector of the corresponding lagrangian contains $N + 1$ hypermultiplets and thus will be modeled by $X(N + 1)$. In order to calculate the masses of the fields on the Higgs branch we need to specify an explicit vector multiplet sector. Here we will use the family of very special real manifolds discussed in appendix B including an arbitrary number $n_V \geq N - 1$ of vector multiplets. Even though these manifolds are not directly related to an exact vector multiplet sector arising from a CY compactification, this ansatz is general enough that it can readily be adjusted to any particular conifold transition by substituting the corresponding vector multiplet sector obtained from eq. (2.34).

4.1 Constructing the action

The construction of the lagrangian describing a general conifold transition proceeds exactly as in the case of the minimal model. Our first task is to generalize the single Killing vector (3.1) to a set of Killing vectors $k_\alpha^X(q)$ which encode the charges of the extra hypermultiplets with respect to the gauge fields A_μ^α associated with the $h^\alpha(\phi)$ parameterizing the volume of the collapsing cycles $C^{(\alpha)}$. The microscopic analysis from subsection 2.3 indicates that the charges are encoded in a set of Killing vectors $\alpha = 1, \ldots, N-1$ of the form

$$\hat{k}_\alpha^v = i\left[0\,,\, v^2\,,\, 0, \ldots,\, -v^{\alpha+2}\,, 0, \ldots, 0\right]^{\mathrm{T}}\,, \tag{4.1}$$

$$\hat{k}_\alpha^u = i\left[0\,,\, -u^2\,,\, 0, \ldots,\, u^{\alpha+2}\,, 0, \ldots, 0\right]^{\mathrm{T}}\,, \tag{4.2}$$

where the entries '$-v^{\alpha+2}$' and '$u^{\alpha+2}$' occur at the $\alpha+2$ position. The components of $\hat{k}_\alpha^X$ with respect to $\partial_{\bar{v}^i}$ and $\partial_{\bar{u}_i}$ are obtained from $\hat{k}_\alpha^v$ and $\hat{k}_\alpha^u$ by complex conjugation. Here we have chosen our hypermultiplets such that v^1, u_1 parameterize the universal hypermultiplet, v^2, u_2 arise from the M2-brane wrapping the cycle $C^{(1)} = -\sum_{\alpha=1}^{N-1} C^{(\alpha+1)}$, and the remaining hypermultiplets $v^{\alpha+2}, u_{\alpha+2}$ correspond to the M2-branes wrapping the cycles $C^{(\alpha+1)}$, $\alpha = 1, \ldots, N-1$, respectively. The set of Killing vectors (4.1) mutually commutes, $[\hat{k}_\alpha, \hat{k}_\beta] = 0$, so that the hypermultiplets are charged with respect to a subgroup $\mathrm{U}(1)^{N-1} \subset \mathrm{U}(1)^{n_V+1}$. Note that the minimal model of the last section is obtained from (4.1) by truncating the universal hypermultiplet (erasing the first slot in the Killing vectors) and restricting to the case $N = 2$.

Our next task is to determine the generators $t_\alpha \in su(N+1, 2)$ which give rise to the isometries (4.1). For this purpose we first restrict the expression for a general Killing vector on $X(N+1)$, (2.12), to the subset of Killing vectors which are linear in the hypermultiplet scalar fields

$$\begin{aligned} \hat{k}^{v^i} &= i\left(t^i{}_j v^j - v^i\, t^{n+1}{}_{n+1}\right), \\ \hat{k}^{u_i} &= i\left(t^{n+1}{}_{n+1} u_i - t^j{}_i u_j - u_i\left(t^{n+1}{}_{n+1} - t^n{}_n\right).\right)\,. \end{aligned} \tag{4.3}$$

Equating this expression with the Killing vectors (4.1) and imposing the condition that t_α is traceless leads to an overdetermined system of equations from which the generators t_α can be determined. In the case of the Killing vectors (4.1) we find the solution

$$t_\alpha = \mathrm{diag}\left[\,0\,,\,1\,,\,0\,,\,\ldots\,,\,-1\,,\,0\,,\,\ldots\,,\,0\,\right], \tag{4.4}$$

where the entry '-1' sits at the $\alpha+2$ position. Hence the vectors (4.1) are indeed triholomorphic isometries of $X(N+1)$.

In the next step we calculate the moment maps $\hat{\mu}^r_\alpha$ of these isometries. Substituting the generators t_α into eq. (2.14) and taking the linear combinations $\hat{\mu}^1 = \hat{\mu}^+ + \hat{\mu}^-$, $\hat{\mu}^2 = -i\,(\hat{\mu}^+ - \hat{\mu}^-)$ we obtain

$$\hat{\mu}^r_\alpha = \begin{bmatrix} -\frac{i}{2\phi_+^{1/2}\phi_-^{1/2}}\,\left(v^2 u_2 - v^{\alpha+2}u_{\alpha+2} - \bar{v}^2\bar{u}_2 + \bar{v}^{\alpha+2}\bar{u}_{\alpha+2}\right) \\ -\frac{1}{2\phi_+^{1/2}\phi_-^{1/2}}\,\left(v^2 u_2 - v^{\alpha+2}u_{\alpha+2} + \bar{v}^2\bar{u}_2 - \bar{v}^{\alpha+2}\bar{u}_{\alpha+2}\right) \\ -\frac{1}{2\phi_-}\left(u_2\bar{u}_2 - u_{\alpha+2}\bar{u}_{\alpha+2}\right) + \frac{1}{2\phi_+}\,\left(v^2\bar{v}^2 - v^{\alpha+2}\bar{v}^{\alpha+2}\right) \end{bmatrix} . \tag{4.5}$$

In order to complete the construction of the lagrangian, we then have to gauge these isometries. Following our previous discussion we need $N-1$ gauge fields A^α_μ whose corresponding scalar fields $h^\alpha(\phi)$ vanish at some subset of the vector multiplet scalar manifold. By virtue of eq. (B.5) this condition is satisfied for all $h^I(\phi)$, $I \geq 1$, but not for $h^0(\phi)$. Without loss of generality we then choose the gauge fields A^α_μ, $\alpha = 1, \ldots, N-1$, to serve as the gauge connections for the Killing vectors.

The gauging of the lagrangian proceeds analogous to the gauging in the minimal model. Again we start by considering the kinetic terms. As in the previous case, we do not gauge any isometries in the vector multiplet sector, so that the corresponding covariant derivative is again given by eq. (3.5). In the hypermultiplet sector we gauge the subgroup $\mathrm{U}(1)^{N-1} \subset \mathrm{U}(1)^{n_V}$. With the choice of gauge connections made above, the covariant derivative in the hypermultiplet kinetic term becomes

$$\mathcal{D}_\mu\, q^X = \partial_\mu\, q^X + \mathrm{g}\, A^\alpha_\mu\, k^X_\alpha(q) \,. \tag{4.6}$$

This expression indicates that the universal hypermultiplet v^1, u_1 is neutral, while the hypermultiplet scalar field v^2 carries charge $q = +1$ with respect to all gauge fields $A^\alpha_\mu, \alpha = 1, \ldots, N-1$ and the fields $v^{\alpha+2}$ have charge $q = -1$ with respect to A^α_μ only. This charge assignment then is in complete agreement with the microscopic description of the transition.

Finally we construct the scalar potential (2.18) of our lagrangian. Evaluating the expressions (2.17) for the particular gauging at hand we obtain

$$P^r = \frac{1}{2}\, h^\alpha(\phi)\, \hat{\mu}^r_\alpha(q) \,, \qquad P^r_x = \frac{1}{2}\, h^\alpha_x(\phi)\, \hat{\mu}^r_\alpha(q) \,, \qquad K^X(\phi, q) = h^\alpha(\phi)\, k^X_\alpha(q) \,, \tag{4.7}$$

with the sets $k^X_\alpha(q)$ and $\hat{\mu}^r_\alpha(q)$ given in (4.1) and (4.5), respectively. The scalar potential of this lagrangian is determined by substituting these relations, the hypermultiplet metric (2.10) and the inverse vector multiplet scalar metric (B.7) into

$$\mathbb{V}(\phi, q) = -4 P^r P^r + 2 g^{xy} P^r_x P^r_y + \frac{3}{4} g_{XY} K^X K^Y \,. \tag{4.8}$$

This completes the construction of the lagrangian. Let us remark that rewriting $\mathbb{V}(\phi, q)$ in terms of the superpotential (2.19) is not helpful in this case as it is simpler to determine the vacuum structure and mass matrices of the lagrangian in terms of the moment maps and the potential (4.8).

4.2 Vacua and mass matrices

We now check whether the supersymmetric vacuum structure and the mass matrices of the lagrangians constructed above agree with the specifications coming from the microscopic theory. Determining the vacuum structure again utilizes the algebraic eqs. (2.23) and proceeds completely analogous to the minimal model. Looking at the set of moment maps (4.5) reveals that they vanish if and only if all charged hypermultiplets have the same value, i.e., if $v^2 = \ldots = v^{N+1} = v$ and $u_2 = \ldots = u_{N+1} = u$. To solve the second condition in (2.23) we then have the choice to either fix $v = u = 0$, leading to the Coulomb branch

$$\mathcal{M}_{\text{Coul}} = \left\{ v^2 = \ldots = v^{N+1} = 0 \, , u_2 = \ldots = u_{N+1} = 0 \right\} , \tag{4.9}$$

or to set $\phi^\alpha = 0 \Leftrightarrow h^\alpha(\phi) = 0$ which corresponds to the Higgs branch

$$\mathcal{M}_{\text{Higgs}} = \left\{ v = v^2 = \ldots = v^{N+1} \, , \, u = u_2 = \ldots = u_{N+1} \, , \, \phi^\alpha = 0 \right\} . \tag{4.10}$$

The scalars not present in these equations are not fixed by the vacuum conditions and correspond to flat directions. We further note that the non-trivial value of the hypermultiplet scalars on the Higgs branch breaks the $\mathrm{U}(1)^{N-1} \subset \mathrm{U}(1)^{n_V+1}$ spontaneously. Substituting the conditions of vanishing $\hat{\mu}^r_\alpha$ and K^X into the potential (4.8) shows that these vacua are Minkowski so that we obtain the correct vacuum structure expected for a conifold transition.

Our next task is to calculate the mass matrices for these vacuum branches. We first analyze the Coulomb branch before turning to the Higgs branch.

In the first step of determining the mass matrix (3.13) we check which terms of the potential (4.8) give rise to a non-zero contribution. Here the first observation is that for the P^r, (4.7), the terms $P^r P^r$ and $g^{xy} P^r_x P^r_y$ are of fourth order in the extra hypermultiplets. This implies that these terms do not contribute to the mass matrix on the Coulomb branch since they vanish identically when taking two derivatives with respect to any scalar field and restricting to $\mathcal{M}_{\text{Coul}}$ afterwards. Hence the masses of our fields are solely generated by the last term in eq. (4.8).

In the next step we show that the vector multiplet scalar fields ϕ^x are massless. The matrix

$$\mathcal{M}_{\Lambda\Sigma} := \partial_\Lambda \partial_\Sigma \left(\frac{3}{4} \, \mathrm{g}^2 \, g_{XY} \, K^X \, K^Y \right) \Big|_{\mathcal{M}_{\text{Coul}}} \tag{4.11}$$

has non-trivial entries if and only if both Λ and Σ take values in the hypermultiplet sector. To see this, we expand $K^X = h^\alpha(\phi) k^X_\alpha(q)$ and note that $k^X_\alpha(q)$ vanishes when restricted to $\mathcal{M}_{\text{Coul}}$. This implies $\mathcal{M}_{\Lambda\Sigma}$ is only non-trivial if there is one derivative acting on each of the Killing vectors $k^X_\alpha(q)$. Since $g^{\Xi\Lambda} = g^{XY} \oplus g^{xy}$ is the direct sum of the hypermultiplet and vector multiplet inverse metrics, we find that non-trivial entries of the mass matrix (3.13) may occur in the hypermultiplet sector only. This establishes that the vector multiplet scalars ϕ^x are massless. In order to determine the masses of the hypermultiplet scalar fields we then restrict ourselves to evaluating the matrix

$$\mathcal{M}_{XY} = \frac{3}{2} \, \mathrm{g}^2 \, h^\alpha \, h^\beta \, g_{WZ} \, \left(\partial_X \, k^W_\alpha \right) \, \left(\partial_Y \, k^Z_\beta \right) \Big|_{\mathcal{M}_{\text{Coul}}} . \tag{4.12}$$

For this purpose we first compute $K^{Y}_{X} := h^\alpha \partial_X k^Y_\alpha(q)$. Using the basis

$$q^X = \left\{ v^1 \,, \dots , v^{N+1} \,, \bar{v}^1 \,, \dots , \bar{v}^{N+1} \,, u_1 \,, \dots , u_{N+1} \,, \bar{u}_1 \,, \dots , \bar{u}_{N+1} \right\} \tag{4.13}$$

the resulting matrix is diagonal,

$$K^{Y}_{X} = i \,\mathrm{diag}\, [\, H \,, -H \,, -H \,, H \,] \,, \tag{4.14}$$

with

$$H = \mathrm{diag}\left[\, 0 \,, -\sum h^\alpha \,, h^1 \,, \dots \,, h^{N-1} \right] . \tag{4.15}$$

Next we restrict the hypermultiplet scalar metric $g_{XY}(q)$ to the Coulomb branch. We find that all blocks appearing in $g_{XY}(q)|_{\mathcal{M}_{\mathrm{Coul}}}$ are diagonal:

$$\begin{aligned}
g_{v_1\bar{v}_1} &= \frac{1}{2\phi_+^2\phi_-^2} \left(1 - \bar{u}_1 u_1 \left(1 - \bar{v}^1 v^1\right)^2\right) , & g_{v^{\alpha+1}\bar{v}^{\beta+1}} &= \frac{1}{2\phi_+}\,\delta_{\alpha+1,\beta+1} \,, \\
g_{u_1\bar{u}_1} &= \frac{1}{2\phi_-^2} \left(1 - \bar{v}^1 v^1\right) , & g_{u_{\alpha+1}\bar{u}_{\beta+1}} &= \frac{1}{2\phi_-}\,\delta^{\alpha+1,\beta+1} \,, \\
g_{\bar{v}^1 u_1} &= -\frac{1}{2\phi_-^2} \left(\bar{u}_1 v^1\right) .
\end{aligned} \tag{4.16}$$

Here and in the following ϕ_+ and ϕ_- are understood to be restricted to $\mathcal{M}_{\mathrm{Coul}}$. The matrix $\mathcal{M}_{XY}$ can now be computed from

$$\mathcal{M}_{XY} = \frac{3}{2}\mathrm{g}^2 \left[K \, g \, K^{\mathrm{T}} \right] . \tag{4.17}$$

Explicitly, we find

$$\mathcal{M}_{XY} = \begin{bmatrix} 0 & A & 0 & 0 \\ A & 0 & 0 & 0 \\ 0 & 0 & 0 & B \\ 0 & 0 & B & 0 \end{bmatrix} , \tag{4.18}$$

with A and B being the following $(N+1) \times (N+1)$-dimensional block matrices:

$$\begin{aligned}
A &= \frac{3}{4\phi_+}\,\mathrm{g}^2 \mathrm{diag}\left[\, 0 \,, \left(\sum h^\alpha\right)^2 , \left(h^1\right)^2 , \dots , \left(h^{N-1}\right)^2 \right] , \\
B &= \frac{3}{4\phi_-}\,\mathrm{g}^2 \mathrm{diag}\left[\, 0 \,, \left(\sum h^\alpha\right)^2 , \left(h^1\right)^2 , \dots , \left(h^{N-1}\right)^2 \right] .
\end{aligned} \tag{4.19}$$

Finally we need to calculate the inverse metric g^{XY}, restricted to $\mathcal{M}_{\mathrm{Coul}}$, by inverting $g_{XY}|_{\mathcal{M}_{\mathrm{Coul}}}$ given in (4.16). The only non-zero components of $g^{XY}|_{\mathcal{M}_{\mathrm{Coul}}}$ are given by

$$\begin{aligned}
g^{v^1\bar{v}^1} &= 2\,\phi_+^2\,\phi_- \,, & g^{v^{\alpha+1}\bar{v}^{\beta+1}} &= 2\phi_+ \delta^{\alpha+1,\beta+1} \,, \\
g^{u_1\bar{u}_1} &= 2\,\frac{\phi_-}{\phi_+} \left(1 - \bar{u}_1 u_1 \left(1 - \bar{v}^1 v^1\right)^2\right) , & g^{u_{\alpha+1}\bar{u}_{\beta+1}} &= 2\,\phi_-\,\delta^{\alpha+1,\beta+1} \,, \\
g^{v^1\bar{u}_1} &= 2\phi_+\,\phi_- \left(\bar{u}_1 v^1\right) .
\end{aligned} \tag{4.20}$$

The hypermultiplet masses are given by the eigenvalues of the mass matrix

$$\mathcal{M}^{X}{}_{Y} = g^{XZ}\,\mathcal{M}_{ZY}\big|_{\mathcal{M}_{\mathrm{Coul}}} . \tag{4.21}$$

Explicitly we find the masses for the universal hypermultiplet v^1, u_1, the hypermultiplet v^2, u_2 arising from the wrapped cycle $C^{(1)} = -\sum_{\alpha=1}^{N-1} C^{(\alpha+1)}$ and the other hypermultiplets $v^{\alpha+2}, u_{\alpha+2}$ associated with the cycles $\mathcal{C}^{(\alpha+1)}$, $\alpha = 1, \ldots, N-1$, of volume h^α to be

$$m_1 = 0\,, \quad m_2 = \sqrt{\frac{3}{2}}\,\mathrm{g} \sum_{\alpha=1}^{N-1} h^\alpha\,, \qquad \text{and} \qquad m_{\alpha+2} = \sqrt{\frac{3}{2}}\,\mathrm{g}\, h^\alpha\,, \tag{4.22}$$

respectively. These masses are in complete agreement with the microscopic theory, which demands that the mass of the hypermultiplets is proportional to the volume of the cycle wrapped by the corresponding M2-brane. Comparing these masses to the BPS mass formula (2.33) fixes g to the value (3.17). This input pins down the remaining freedom of the extended LEEA in terms of the microscopic data.

Let us now turn to the Higgs branch. Here we do not have a prediction for the masses of the massive states in terms of the microscopic description. But since the data on the Coulomb branch together with the charge assignment (4.1) completely determines the lagrangian, we can use our model to predict the masses on this branch.

We again begin our calculation by investigating which terms of the potential (4.8) contribute to the mass matrix on the Higgs branch. The first term proportional to $P^r P^r$ is of second order in both ϕ^α and $\hat{\mu}^r_\alpha$ which both vanish when restricted to the Higgs branch. Hence all its second derivatives vanish when restricted to $\mathcal{M}_{\text{Higgs}}$ and there is no contribution to the mass matrix. The second term, $2g^{xy} P^r_x P^r_y$, has orders zero and two in ϕ^α and $\hat{\mu}^r_\alpha$, respectively. This indicates that this term is of relevance for the hypermultiplet mass matrix, as the only non-vanishing second derivative terms (restricted to $\mathcal{M}_{\text{Higgs}}$) are obtained when one derivative with respect to a hypermultiplet scalar acts on each of the moment maps. Finally the third term is of second order in ϕ^α while the Killing vectors k^X_α do not vanish on $\mathcal{M}_{\text{Higgs}}$. Therefore this term gives rise to the vector multiplet scalar masses, as we have to take two derivatives with respect to the vector multiplet scalars to have a non-vanishing expression when restricting to $\mathcal{M}_{\text{Higgs}}$.

The analysis of the supersymmetry representations [38] and the explicit example of our minimal lagrangian show that the massive modes on this branch combine into massive long vector multiplets composed of one vector and one hypermultiplet of the same mass. In order to determine this mass it suffices to compute the vector multiplet scalar masses arising from the third term in the potential.

We start by evaluating

$$\mathcal{M}_{xy} = \frac{3}{2}\,\mathrm{g}^2\, g_{XY} \left(\partial_x\, h^\alpha(\phi)\, k^X_\alpha\right) \left(\partial_y\, h^\beta(\phi)\, k^Y_\beta\right)\Big|_{\mathcal{M}_{\text{Higgs}}}\,. \tag{4.23}$$

To this end we first calculate the $(n_V) \times 4(N+1)$ dimensional matrix $K_x{}^X := \partial_x\, h^\alpha(\phi)\, k^X_\alpha$. Taking the basis (4.13) for the hypermultiplets and using the definition (B.5) for $h^I(\phi)$ we find

$$K_x{}^X = h^0 \begin{bmatrix} -iv^2\mathcal{Q} & i\bar{v}^2\mathcal{Q} & iu_2\mathcal{Q} & -i\bar{u}_2\mathcal{Q} \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{4.24}$$

where $\mathcal{Q}$ denotes the $(N-1)\times(N+1)$ matrix[24]

$$\mathcal{Q} = \left[\begin{array}{cc|c} 0 & -1 & \\ \vdots & \vdots & \mathbb{1}_{(N-1)\times(N-1)} \\ 0 & -1 & \end{array}\right], \tag{4.25}$$

and '0' is short for the $(n_V - N + 1)\times(N+1)$ dimensional zero matrix.

It is then straightforward to calculate $\mathcal{M}_{xy} = \frac{3}{2}\,\mathrm{g}^2\,[K\,g\,K^{\mathrm{T}}]_{xy}\,|_{\mathcal{M}_{\mathrm{Higgs}}}$ to be

$$\mathcal{M}_{xy} = 3\,\mathrm{g}^2\,(h^0)^2\,\mathrm{diag}\,[\,\Pi\,,\,0\,]\,. \tag{4.26}$$

Here Π denotes the $(N-1)\times(N-1)$ matrix

$$\Pi = |v|^2\,\mathcal{Q}g_{v\bar{v}}\mathcal{Q}^{\mathrm{T}} + |u|^2\,\mathcal{Q}g_{u\bar{u}}\mathcal{Q}^{\mathrm{T}} - v\bar{u}\,\mathcal{Q}g_{\bar{u}v}\mathcal{Q}^{\mathrm{T}} - u\bar{v}\,\mathcal{Q}g_{u\bar{v}}\mathcal{Q}^{\mathrm{T}}\,, \tag{4.27}$$

where v, u are the values of the hypermultiplet scalars on the Higgs branch and $g_{\bar{v}v}$, $g_{\bar{u}u}$, $g_{\bar{u}v}$ and $g_{u\bar{v}}$ denote the block matrices appearing in the hypermultiplet scalar metric $g_{XY}(q)$. Introducing $M := \mathcal{Q}g\mathcal{Q}^{\mathrm{T}}$ with g representing the blocks of $g_{XY}(q)$ we find that there is a simple relation between the components of M and g

$$m_{\alpha\beta} = g_{22} - g_{(\alpha+2)2} - g_{2(\beta+2)} + g_{(\alpha+2)(\beta+2)}\,. \tag{4.28}$$

Evaluating this expression on $\mathcal{M}_{\mathrm{Higgs}}$, we find $M = 0$ if $g = g_{u\bar{v}}$ or $g = g_{\bar{u}v}$ while for $g = g_{v\bar{v}}$ and $g = g_{u\bar{u}}$ we obtain

$$m_{\alpha\beta} = \frac{1}{2\phi_+}\,(1+\delta_{\alpha\beta}) \qquad \text{and} \qquad m_{\alpha\beta} = \frac{1}{2\phi_-}\,(1+\delta_{\alpha\beta})\,, \tag{4.29}$$

respectively. This allows us to write down $\mathcal{M}_{xy}$ explicitly. Its non-zero components read

$$\mathcal{M}_{\alpha\beta} = \frac{3}{2}\,\mathrm{g}^2\,(h^0)^2\left(\frac{1}{\phi_+}\,|v|^2 + \frac{1}{\phi_-}\,|u|^2\right)(1+\delta_{\alpha\beta})\,. \tag{4.30}$$

The mass matrix $\mathcal{M}^x{}_y$ for the vector multiplet scalars is obtained by raising one index of $\mathcal{M}_{xy}$ using the inverse vector multiplet scalar metric restricted to the Higgs branch (B.8). Defining

$$(m_{\mathrm{Higgs}})^2 := \mathrm{g}^2\left((h^0)^2 - \frac{3}{2}\sum_{\sigma=N}^{n_V}(h^\sigma)^2\right)\left(\frac{1}{\phi_+}|v|^2 + \frac{1}{\phi_-}\,|u|^2\right), \tag{4.31}$$

where ϕ_+ and ϕ_- are understood to be restricted to $\mathcal{M}_{\mathrm{Higgs}}$, we find

$$\mathcal{M}^x{}_y = (m_{\mathrm{Higgs}})^2\,\mathrm{diag}\left[\mathcal{M}^\alpha{}_\beta\,,\,0\right], \tag{4.32}$$

with $\mathcal{M}^\alpha{}_\beta = 1 + \delta^\alpha{}_\beta$.

This result shows that all 'spectator vector multiplets' $h^\sigma, \sigma = N,\ldots,n_V$, which do not serve as a gauge connection for the isometries remain massless. Determining the eigenvalues of $\mathcal{M}^\alpha{}_\beta$ shows that the vector multiplet scalars ϕ^α acquire two different masses. One scalar

[24] This matrix is the charge matrix of the $N+1$ hypermultiplets with respect to the $\mathrm{U}(1)^{N-1}$ gauge connections A^α_μ.

has mass $(N)^{1/2}\, m_{\text{Higgs}}$ while the remaining $N-2$ scalars have degenerate mass m_{Higgs}. Here it is interesting to observe that, in the presence of spectator vector multiplets, the mass of these states depends on both the vector and hypermultiplet degrees of freedom. This points at the non-BPS nature of the corresponding long vector multiplets. We also observe that in the absence of spectator vector multiplets and for $N = 2$ the general formula (4.32) agrees with the results obtained for the minimal lagrangian of the previous subsection.

In terms of CY compactifications, these general lagrangians have the following interpretation. On the Coulomb branch, associated with the CY compactification on X, we have n_V vector and one hypermultiplet which are generically massless. At the conifold locus one obtains N additional massless hypermultiplets. When going to the Higgs branch arising from the CY compactification on $\tilde{X}$, $N-1$ charged hypermultiplets are 'eaten up' by the massive vector multiplets to form long vector multiplets. We are then left with $n_V-(N-1)$ vector and two hypermultiplets which are generically massless. This establishes that our extended LEEA indeed provides a continuous description of a conifold transition where the Hodge numbers change by $h^{1,1}(\tilde{X}) = h^{1,1}(X) - (N-1)$ and $h^{1,2}(\tilde{X}) = h^{1,2}(X) + 1$, while the Euler number changes by $\chi_E(\tilde{X}) = \chi_E(X) - 2N$. In particular, this class includes the conifold transition of the quintic studied in [9, 40] where the Hodge numbers of X and $\tilde{X}$ are given by $h^{1,1}(X) = 102$, $h^{1,2}(X) = 0$ and $h^{1,1}(\tilde{X}) = 87$, $h^{1,2}(\tilde{X}) = 1$.

5. Conclusions

In this paper we have constructed gauged supergravity actions which model the LEEA of M-theory compactified on a CY threefold undergoing a conifold transition. These actions explicitly include the extra states descending from wrapped M2-branes and have a non-trivial scalar potential with a Coulomb and a Higgs branch. While their vector multiplet sector was determined exactly, we made a particular choice for the hypermultiplet scalar metrics in order to obtain explicit actions. In a companion paper [16] we use these actions to investigate the M-theory physics of conifold transitions, in particular the dynamical realization of conifold transitions and the moduli dynamics in cosmological solutions.

Since the explicit actions constructed in this paper only cover the case where an arbitrary number of two-cycles satisfying $r = 1$ homology relations are contracted, more work has to be done to extend our construction to the case of general r. Ultimately one would of course like to derive the metrics on the scalar manifolds from M-theory. Presumably, the exact effective action $\hat{S}$ is determined by combining geometrical data of the singular space $\hat{X}$ with data involving the M2-branes wrapping the vanishing cycles. The extended LEEA found in this paper suggest that this combined set of data is meaningful and in some sense smooth.

To explain what we mean let us elaborate on the discussion of the relation between threshold corrections and geometry in [13]. Recall that the vector multiplet sector can be treated exactly[25] and assigns coefficients C_{IJK}, $\hat{C}_{IJK}$, $\tilde{C}_{\tilde{I}\tilde{J}\tilde{K}}$ to the spaces $X, \hat{X}, \tilde{X}$. Since

[25] In section 4 we restricted the vector multiplet manifold to be of a special type, but only in order to be able to compute the mass matrix explicitly.

$C_{IJK}, \tilde{C}_{\tilde{I}\tilde{J}\tilde{K}}$ are the triple intersection numbers of X and $\tilde{X}$, we expect that $\hat{C}_{IJK}$ can be interpreted as triple intersection numbers of the singular space $\hat{X}$. Also note that the combined web of moduli spaces of CY manifolds $X, \tilde{X}$ related by conifold transitions is not a manifold, but only a stratification, because its dimension jumps at the transition point. We can, however, construct the extended LEEA $\hat{X}$, associated with the geometrical quantities of the singular space $\hat{X}$ to obtain a smooth description of the physics at the transition locus where the actions S and $\tilde{S}$ are discontinuous. Thus it seems that M-theory implicitly gives us a prescription of how to resolve the intersection points of the web of moduli spaces, and how to assign regular data to the corresponding singular spaces. This observation is not restricted to conifold transitions, but also applies to flop transitions as well as extremal transition [13] which involve non-abelian gauge symmetry enhancement. It would be very interesting to make these ideas mathematically precise.

Another possible extension of our work is to consider the four-dimensional LEEA of type II string theories on CY threefolds. While essentially nothing changes in the hypermultiplet sector, the vector multiplet sector gets more complicated, because it is now governed by a holomorphic prepotential, instead of a real cubic one. Moreover, the threshold corrections of the vector multiplet couplings resulting from integrating out charged multiplets show a different behavior in five and in four dimensions. As we have seen in section 2, the five-dimensional threshold correction only induce discontinuities in the third derivatives of the prepotential. In four dimensions, however, second derivatives of the prepotential (gauge couplings) become logarithmically divergent as a function of the modulus controlling the distance to the conifold point [8], while its third derivatives (the 'Yukawa couplings') have a first order pole. However, we expect that it is possible to handle these threshold corrections along the lines of [14], where it was shown that one can 'integrate in' four-dimensional vector multiplets. It would be interesting to extend this work to the case where one also integrates in hypermultiplets. Furthermore, conifold transitions are special cases of extremal transitions [41]. In the full class one also finds transitions where one does not have isolated singular points, but curves of singularities. This leads to extra massless charged vector multiplets, and, hence, to the un-Higgsing of non-abelian gauge groups, which could also be phenomenologically interesting.

Another line of research extending the present setup would be the inclusion of background fluxes. Fluxes could in particular lift the still existing flat directions and break the still existing supersymmetry spontaneously. A particular interesting idea is that the interplay between flux and vanishing cycles can generate a hierarchically small scale, and thus might help to solve the gauge hierarchy and the cosmological constant problem [42]. It is also known that when fluxes a turned on, the remaining vacua prefer to sit at special points in moduli space, such as conifold points [8, 43]. Such points could be investigated by including fluxes into our framework.

Also note that the deformed and the resolved conifold geometry can be obtained, at least locally, as solutions of gauged supergravity theories [44], where the gauging results from wrapping branes on cycles or switching on background fluxes. These gaugings are different from those considered in this paper: they do not correspond to integrating in charged multiplets, they are non-compact rather than compact, they lift directions in scalar

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field space, which in our setup are necessarily flat, and they can break supersymmetry spontaneously. This reflects the different roles played by branes: while in [44] branes (or the flux they resolve into) are part of the background, we have included certain light brane degrees of freedom as dynamical degrees of freedom into the lagrangian. It would be interesting to better understand the relation between these complementary setups.

Finally, we would like to step outside the framework of type II CY compactifications (and their deformations by fluxes) and investigate topological phase transitions in genuine $\mathcal{N} = 1$ compactifications such as CY compactifications of type I and heterotic strings, heterotic M-theory, type II CY orientifolds and M-theory compactifications on sevenfolds with G_2 structure.[26] Here topological phase transitions, for example conifold transitions and small instanton transitions, can change the chiral matter content [46]. Part of the LEEA technology applies to these cases as well. In particular the computation of threshold corrections resulting from integrating out states is straightforward, at least at the one loop level, see for example [47]. Of course, when considering LEEA with $\mathcal{N} = 1$ supersymmetry one seems to loose the powerful tool of special geometry. However, there are indications that in string compactifications some features of special geometry survive, so that at least holomorphic quantities are accessible [48].

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A. Vector multiplet sector I: one multiplet

The simplest extended LEEA describing a conifold transition contains two charged hypermultiplets and was constructed in section 4. The vector multiplet sector of this action consists of a single vector multiplet. For convenience we collect all the relevant properties of this sector in this appendix.

Following [49], the prepotential (2.3) of the most general one vector multiplet scalar manifold can be brought to the standard form

$$\mathcal{V} = (h^0)^3 - \frac{3}{2}\, h^0\, (h^1)^2 + c\, (h^1)^3 = 1\,, \tag{A.1}$$

where c denotes an arbitrary real constant. The value $c = -\frac{1}{\sqrt{2}}$ corresponds to the manifold $O(1,1)$ considered in [24] while the prepotential

$$\mathcal{V} = \frac{3}{8} U^3 + \frac{1}{2} U\, T^2 = 1\,, \tag{A.2}$$

which arises from Calabi-Yau compactifications on $T^2 \times K3$ [50, 51] can be mapped to the case $|c| > \frac{1}{\sqrt{2}}$.

[26] See for example [45] for a recent review of the role of singularities and topology change in G_2 compactifications of M-theory.

The metric coupling the vector field kinetic terms is obtained from eq. (2.4) and reads

$$a_{IJ} = \begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix} , \tag{A.3}$$

with entries

$$\begin{aligned} a_{00} &= -2h^0 + 3 \left((h^0)^2 - \frac{1}{2} (h^1)^2 \right)^2 , \\ a_{01} &= h^1 - \frac{3\,h^1}{2} \left(2\,(h^0)^2 - (h^1)^2 \right) \left(h^0 - c\,h^1 \right) , \\ a_{11} &= h^0 - 2\,c\,h^1 + 3\,\left(h^0\,h^1 - c\,(h^1)^2 \right)^2 . \end{aligned} \tag{A.4}$$

Observe that at $h^0 = 1$, $h^1 = 0$, which we interprete as a conifold point in section 3, this becomes the two-by-two unit matrix. Hence the two vector field kinetic terms decouple and have standard normalization at this point.

To fully specify the action, we also need a parameterization of the vector multiplet scalar manifold. Here it is convenient to introduce the vector multiplet scalar field ϕ as coordinate

$$\phi = \frac{h^1}{h^0} . \tag{A.5}$$

Solving this relation for h^1 and substituting the resulting expression into the prepotential (A.1), it is straightforward to calculate $h^I(\phi)$ introduced in eq. (2.3):

$$h^0(\phi) = \left(1 - \frac{3}{2}\,\phi^2 + c\,\phi^3 \right)^{-1/3} , \quad h^1(\phi) = \phi \left(1 - \frac{3}{2}\,\phi^2 + c\,\phi^3 \right)^{-1/3} . \tag{A.6}$$

The vector multiplet scalar manifold (2.6) is then given by

$$g_{xy} = \frac{3\left(2 - 4\,c\,\phi + \phi^2\right)}{\left(2 - 3\,\phi^2 + 2\,c\,\phi^3\right)^2} . \tag{A.7}$$

At the conifold point $h^1(\phi) = 0 \leftrightarrow \phi = 0$ this metric is regular $g_{xy}|_{\phi=0} = \frac{3}{2}$. The coordinate range of ϕ is bounded by the values $\phi_-^{\rm crit}$ and $\phi_+^{\rm crit}$, at which the metric becomes zero or infinite. For $|c| < \frac{1}{\sqrt{2}}$ the metric diverges at both $\phi_-^{\rm crit}$ and $\phi_+^{\rm crit}$ while for $|c| \geq \frac{1}{\sqrt{2}}$ we encounter one zero and one infinity bounding the coordinate range of ϕ. For the special case $c = 0$ considered in [16] we have $\phi_\pm^{\rm crit} = \pm\sqrt{\frac{2}{3}}$ implying the corresponding vector multiplet scalar manifold is given by the interval $]-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}[$. Note that even though the coordinate length of this interval is finite, the boundaries $\phi_\pm^{\rm crit}$ are at infinite geodesic distance from the origin.

B. Vector multiplet sector II: n_V multiplets

The construction of the extended LEEA describing a general conifold transition in section 4 required specifying an explicit vector multiplet sector containing n_V vector multiplets. In this appendix we derive the properties of the vector fields and vector multiplet scalar metrics needed in this construction. The calculation is analogous to the one vector multiplet case considered in appendix A, but due to the increased number of vector multiplets somewhat more involved.

We start by specifying the prepotential (2.3) to be

$$\mathcal{V} = (h^0)^3 - \frac{3}{2}\, h^0 \sum_{x=1}^{n_V} (h^x)^2 = 1\,, \tag{B.1}$$

where $x, y, z = 1, \ldots, n_V$ counts the vector multiplets. The prepotential is again given in the standard form [49], but is not the most general one, as we have chosen $C_{xyz} = 0$ in order to keep the calculation manageable.

The metric coupling the kinetic terms of the vector fields is obtained from eq. (2.4)

$$a_{IJ} = \begin{bmatrix} a_{00} & a_{0x} \\ a_{x0} & a_{xy} \end{bmatrix}\,, \tag{B.2}$$

with the entries being

$$\begin{aligned} a_{00} &= -2h^0 + 3\left((h^0)^2 - \frac{1}{2}\sum_{z=1}^{n_V}(h^z)^2\right)^2\,, \\ a_{0x} &= h^x - 3\,h^0\,h^x\left((h^0)^2 - \frac{1}{2}\sum_{z=1}^{n_V}(h^z)^2\right)\,, \\ a_{xy} &= h^0\,\delta_{xy} + 3\,(h^0)^2\,h^x\,h^y\,. \end{aligned} \tag{B.3}$$

In order to have an explicit parameterization of the vector multiplet scalar manifold we introduce homogeneous coordinates

$$\phi^x = \frac{h^x}{h^0}\,, \qquad x = 1, \ldots, n_V\,. \tag{B.4}$$

Solving these relations for h^x and substituting the resulting expressions into the prepotential (B.1) the h^I become functions of the vector multiplet scalars ϕ^x:

$$h^0(\phi) = \left(1 - \frac{3}{2}\sum_{z=1}^{n_V}(\phi^z)^2\right)^{-1/3}\,, \qquad h^x(\phi) = \phi^x\left(1 - \frac{3}{2}\sum_{z=1}^{n_V}(\phi^z)^2\right)^{-1/3}\,. \tag{B.5}$$

These relations are used to calculate the components of the vector multiplet scalar metric (2.6)

$$g_{xy} = \begin{cases} \dfrac{3\,\left(1 + \frac{1}{2}(\phi^x)^2 - \frac{3}{2}\sum_{z\neq x}^{n_V}(\phi^z)^2\right)}{2\,\left(1 - \frac{3}{2}\sum_{z=1}^{n_V}(\phi^z)^2\right)^2}\,, & \text{for } x = y\,, \\ \dfrac{6\,\phi^x\,\phi^y}{\left(1 - \frac{3}{2}\sum_{z=1}^{n_V}(\phi^z)^2\right)^2}\,, & \text{for } x \neq y\,. \end{cases} \tag{B.6}$$

In order to find the mass matrix on the Higgs branch we further need the inverse vector multiplet scalar metric g^{xy} whose components are given by

$$g^{xy} = \begin{cases} \dfrac{2\left(1-\frac{3}{2}(\phi^x)^2+\frac{1}{2}\sum_{z\neq x}^{n_V}(\phi^z)^2\right)\left(1-\frac{3}{2}\sum_{z=1}^{n_V}(\phi^z)^2\right)}{3\left(1+\frac{1}{2}\sum_{z=1}^{n_V}(\phi^z)^2\right)}, & \text{for } x=y\,, \\ -\dfrac{4\,\phi^x\,\phi^y\left(1-\frac{3}{2}\sum_{z=1}^{n_V}(\phi^z)^2\right)}{3\left(1+\frac{1}{2}\sum_{z=1}^{n_V}(\phi^z)^2\right)}, & \text{for } x\neq y\,. \end{cases} \tag{B.7}$$

The vacuum conditions on the Higgs branch generically fix a subset $\phi^\alpha, \alpha = 1, \ldots, N - r \leq n_V$, to be zero (cf. eq. (4.10) for the case $r = 1$). By virtue of the relation (B.5) this corresponds to the h^α parameterizing the volume of the contracting cycles being zero. Restricting g^{xy} given above to $\phi^\alpha = 0$, the resulting expression becomes block diagonal

$$g^{xy}|_{\phi^\alpha=0} = \begin{bmatrix} a\,\mathbb{1}_{(N-r)\times(N-r)} & 0 \\ 0 & g \end{bmatrix}, \tag{B.8}$$

where

$$a := \frac{2}{3}\left(1-\frac{3}{2}\sum_{\sigma=m+1}^{n_V}(\phi^\sigma)^2\right), \tag{B.9}$$

and g is to be understood as $g^{\sigma_1\sigma_2}|_{\phi^\alpha=0}$ with $\sigma_1, \sigma_2 = N - r + 1, \ldots, n_V$. This data completely determines the vector multiplet sector of the conifold model discussed in section 4 and enables us to explicitly calculate the vector multiplet scalar masses on the Higgs branch.

C. An explicit conifold transition

In this appendix, we give the complete geometrical data for a particular pair of CY manifolds X and $\tilde{X}$ which are related by a conifold transition. The discussion follows [6] where further details can be found.

Our sample CY manifolds $X, \tilde{X}$ are so-called complete intersecting Calabi-Yau manifolds (CICY) [52, 6]. These are special in the sense that they can be obtained as complex submanifolds of an embedding space χ which is given as the product of m complex projective spaces $\mathbb{P}_r^{n_r}$, $r = 1, \ldots, m$,

$$\chi = \mathbb{P}_1^{n_1} \times \cdots \times \mathbb{P}_m^{n_m}\,. \tag{C.1}$$

The submanifold is defined by the intersection of the zero loci of K polynomials, which are homogeneous with respect to the coordinates of the different $\mathbb{P}_m^{n_m}$-factors. Generally CICY are characterized by a configuration matrix

$$\mathcal{M} \in \left[\mathbf{n}\|\mathbf{q}\right] = \left[\begin{array}{c||ccc} n_1 & q_1^1 & \ldots & q_K^1 \\ \vdots & \vdots & & \vdots \\ n_m & q_1^m & \ldots & q_K^m \end{array}\right] \tag{C.2}$$

with the n_r being the dimension of $\mathbb{P}_r^{n_r}$ in (C.1) and the integers q_a^r, $a = 1, \ldots, K$ denoting the degree of the K-th constraint polynomial with respect to the homogeneous coordinates of $\mathbb{P}_r^{n_r}$.

Introducing J_r as the Kähler form of $\mathbb{P}_r^{n_r}$ the Euler characteristic of the manifold $\mathcal{M}$ can be computed by evaluating the relation [6]

$$\left[\frac{\prod_{r=1}^{m}(1+J_r)^{n_r+1}}{\prod_{a=1}^{K}\left(1+\sum_{r=1}^{m} q_a^r\, J_r\right)}\right]\cdot \prod_{a=1}^{K}\left(\sum_{r=1}^{m} q_a^r\, J_r\right) = \cdots + \chi_{\mathrm{E}}(\mathcal{M})\cdot \prod_{r=1}^{m}(J_r)^{n_r} \tag{C.3}$$

by expanding the l.h.s. in J_r and evaluating the coefficient of the term $\prod_{r=1}^{m}(J_r)^{n_r}$.

As a concrete example we consider the families of CY X and $\tilde{X}$ given by the configurations

$$X \in \left[\begin{array}{c\|ccc} 5 & 4 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{array}\right]^{2}_{-168}, \qquad \tilde{X} \in \left[\begin{array}{c\|cc} 5 & 4 & 2 \end{array}\right]^{1}_{-176}. \tag{C.4}$$

Here the subscript denotes the Euler number χ_E of the configuration computed from eq. (C.3) while the superscript gives its Hodge number $h^{1,1}$ which is taken from [6].

For the configuration X the embedding space is $\chi = \mathbb{P}_1^5 \times \mathbb{P}_2^1$. Taking the homogeneous coordinates on χ and $\mathbb{P}_1^5 \times \mathbb{P}_2^1$ to be x^α, $\alpha = 1, \ldots, 6$ and y^β, $\beta = 1, 2$ the three constraint polynomials can be written as

$$Q(x) = 0\,, \qquad y^1\, x^1 + y^2\, x^2 = 0\,, \qquad y^1\, x^3 + y^2\, x^4 = 0\,, \tag{C.5}$$

where $Q(x)$ denotes some quadric polynomial in the coordinates x^α. Explicitly, we will use $Q(x) = \sum_{\alpha=1}^{6}(x^\alpha)^4$ for which (C.5) becomes a set of transverse polynomials. Let us now consider the last two constraint equations in (C.5). As y^1 and y^2 are homogeneous coordinates in $\mathbb{P}_2^1$, they cannot vanish simultaneously. Satisfying these equations then requires that the determinant

$$C(x) = x^1\, x^4 - x^2\, x^3 = 0 \tag{C.6}$$

vanishes. This constraint has a double zero at the points $x^1 = x^2 = x^3 = x^4 = 0$ indicating that the gradient vanishes at these points, $\mathrm{d}C(x) = 0$. This shows that if we shrink the radius of the P_2^1 in $\chi = \mathbb{P}_1^5 \times \mathbb{P}_2^1$ in (C.5) to zero we obtain a singular CY space $\hat{X}$. Substituting $x^1 = x^2 = x^3 = x^4 = 0$ into the first constraint $Q(x) = 0$

$$Q(x) = \sum_{\alpha=1}^{6}(x^\alpha)^4\,, \quad (0,0,0,0,\omega,1) \tag{C.7}$$

we find that $\hat{X}$ develops four isolated nodes corresponding to the four solutions of $\omega^4 = -1$. Hence, by shrinking the $\mathbb{P}_2^1$ we obtain a singular CY $\hat{X}$ where $N = 4$ holomorphic curves have collapsed to points.

These nodes can be smoothed by adding a quadratic polynomial $t \cdot r(x)$ to (C.6) which does not vanish at the nodes of $\tilde{X}$. The corresponding desingularized CY manifold is given by the zero locus of the quadric and quadratic polynomials

$$Q(x) = 0\,, \qquad C(x) + t\, r(x) = 0\,, \tag{C.8}$$

in the embedding space $\chi = \mathbb{P}_1^5$, corresponding to the configuration $\tilde{X}$ in (C.4).[27]

[27]In the mathematical language X and $\tilde{X}$ are said to be related by a operation called determinant splitting [52, 1, 2] which connects all CICY [3, 2].

From the discussion above, we see that the moduli spaces of X and $\tilde{X}$ share some common points where X and $\tilde{X}$ develop nodes. The difference in the Euler characteristics (C.4) indicates that the topological transition connecting X and $\tilde{X}$ has to be a conifold transition. Substituting the Hodge numbers $h^{1,1}(X) = 2$ and $h^{1,1}(\tilde{X}) = 1$ into the relation (2.32) we obtain, that the $N = 4$ contracting curves $\mathcal{C}_i$ (which are in the same homology class) satisfy $r = 3$ homology relations. Hence the configurations (C.4) are related by a conifold transition with $N = 4$, $r = 3$. Since mirror symmetry exchanges the roles of even-dimensional and odd-dimensional cycles, and of Kähler and complex structure moduli, the corresponding mirror manifolds are related by a conifold transition with $N = 4$, $r = 1$.

Our particular example further has the advantage that it is fairly simple to calculate the triple intersection numbers C_{IJK} of the configurations (C.4). In this context we note that the Lefschetz hyperplane theorem (see [6] for details) applies to X, $\tilde{X}$. This guarantees that the $H^{1,1}$-cohomology is generated by the pullback of the Kähler forms J_r of the $\mathbb{P}^{n_r}_r$. The Kähler form $J_r \in H^{1,1}(\mathbb{P}^{n_r}_r)$ may then be represented by adding a linear constraint on $\mathbb{P}^{n_r}_r$ to the configuration which is independent of the other coordinates in the other factors in χ. In fact, it can be seen from eq. (C.3) that adding a linear constraint to a configuration (C.2) results in an additional factor J_r to the LHS product in (C.3), so that the triple intersection numbers[28]

$$C_{rst} = \int_{\mathcal{M}} J_r\, J_s\, J_t = \int_{\chi} J_r\, J_s\, J_t \wedge \prod_{a=1}^{K} \left(\sum_{r=1}^{m} q^r_a\, J_r \right) \tag{C.9}$$

can be obtained by adding three linear constraints to the configuration matrix (C.2) and then evaluating (C.3) for this extended configuration. For the particular configuration X we represent the constraints corresponding to J_1 and J_2 by

$$X_0 := \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad X_1 := \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{C.10}$$

respectively. For $\tilde{X}$ the constraint X_1 drops out as this constraint is associated with the factor $\mathbb{P}^1_2$ which vanishes on $\tilde{X}$.

Thus the C_{IJK} of the configurations X and $\tilde{X}$ are then be computed as

$$C_{IJK} = \chi_E \left[\chi \middle\| \mathbf{q}\; X_I\; X_J\; X_K \right]. \tag{C.11}$$

Explicitly, we find

$$\begin{aligned} &C_{000} = 8\,, \qquad C_{001} = 4\,, \qquad C_{011} = 0\,, \qquad C_{111} = 0\,, \\ &\tilde{C}_{000} = 8\,. \end{aligned} \tag{C.12}$$

This implies that the prepotentials of X and $\tilde{X}$ are given by

$$\mathcal{V} = 8\,(h^0)^3 + 12\,(h^0)^2\,(h^1) = 1\,, \qquad \text{and} \tag{C.13}$$

$$\tilde{\mathcal{V}} = 8\,(h^0)^3 = 1\,, \tag{C.14}$$

[28] Note that the factor $\prod_{a=1}^{K} \left(\sum_{r=1}^{m} q^r_a\, J_r \right)$ works like a delta function, restricting the integration on the embedding space χ to the submanifold $\mathcal{M}$.

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respectively. Here we see that $\mathcal{V}$ and $\tilde{\mathcal{V}}$ are indeed related by the truncation rule given in subsection 2.3 which, in the case at hand corresponds to setting the volume of the shrinking cycle, h^1, to zero.

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